JACK ON A DEVIL'S STAIRCASE

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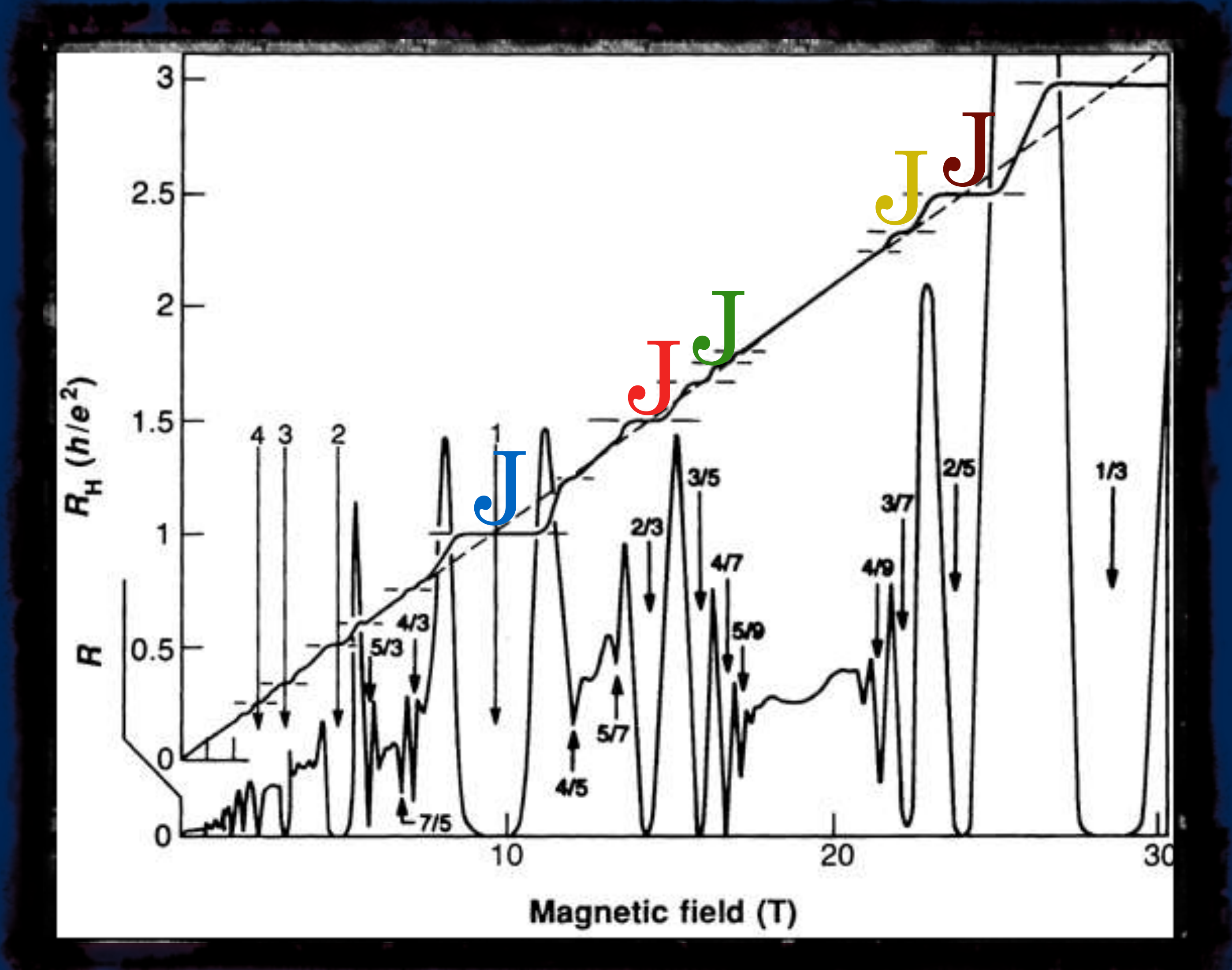

(on the ground states of the FQHE)

28 GIUGNO 2017 - MILANO

Devil’s staircase phase diagram of the FQHE in the thin torus limit

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Unified Fock space representation of FQH states

PRB 95 (2017) 245123

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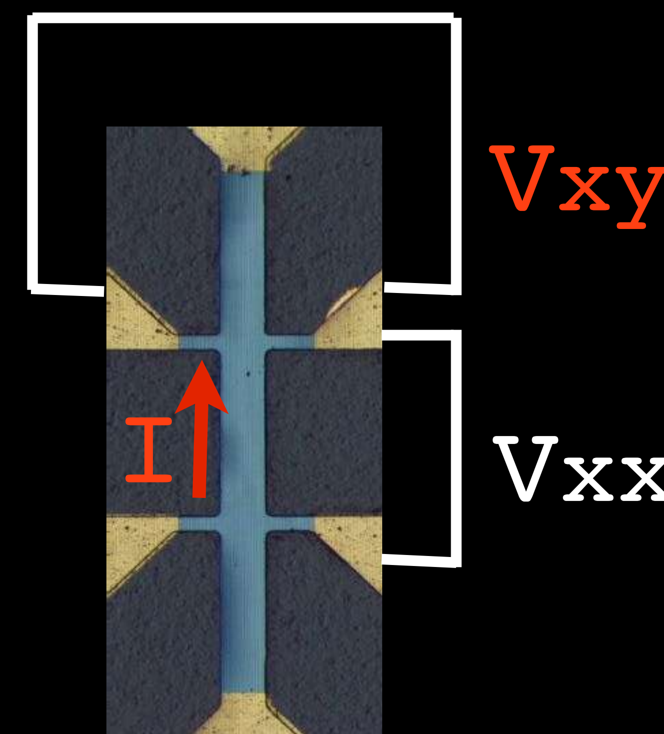
THE HALL EFFECT

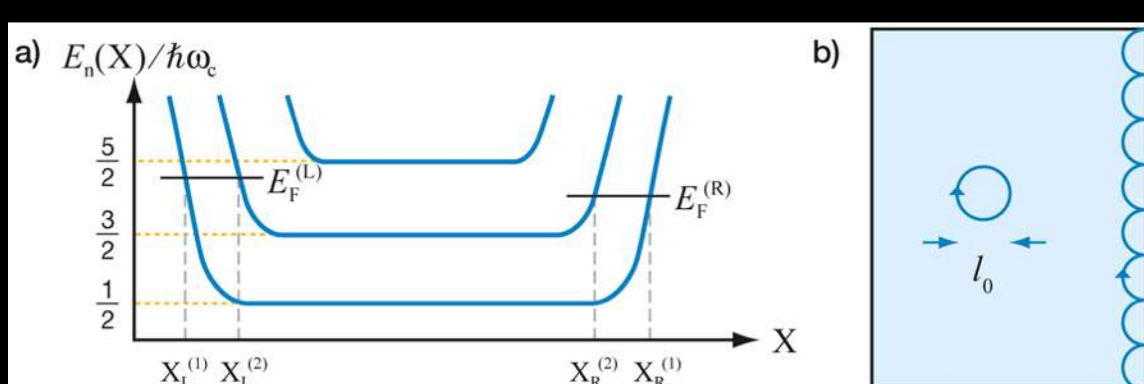
classical

Rxy = Vxy/I = B/nec



2D electron gas
T ~ 100 mK or less
B ~ tens of Tesla

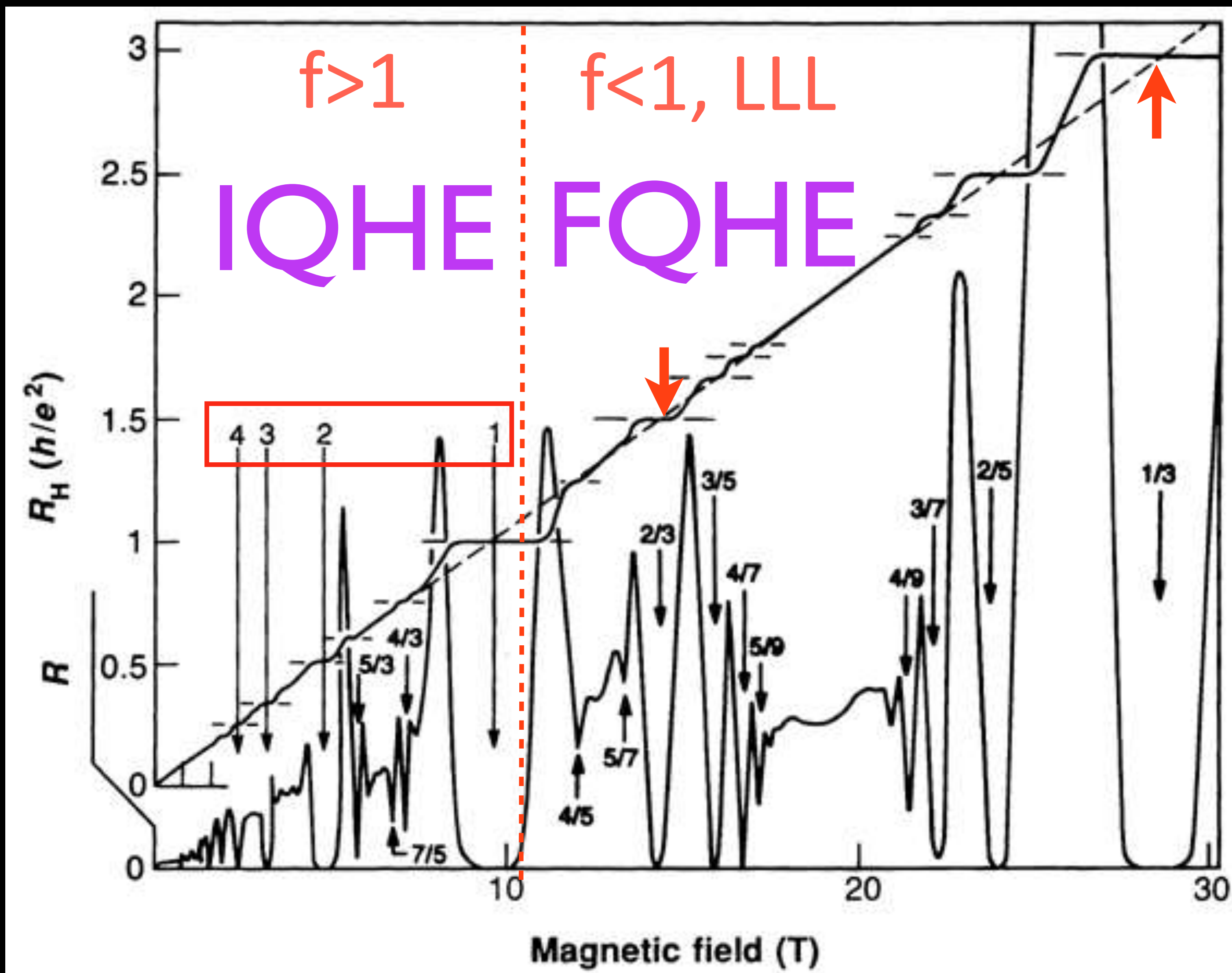
quantum



Rxy = 1/f [h/e^2]
f = filling fraction
= # electrons/degen.LL

IQHE (1980)

Klaus von Klitzing
Max Planck
Nobel 1985



FQHE (1981)

Horst Stormer
Columbia

Nobel 1988
For their discovery of a new form of quantum fluid with fractionally charged excitations

Daniel Tsui
Princeton

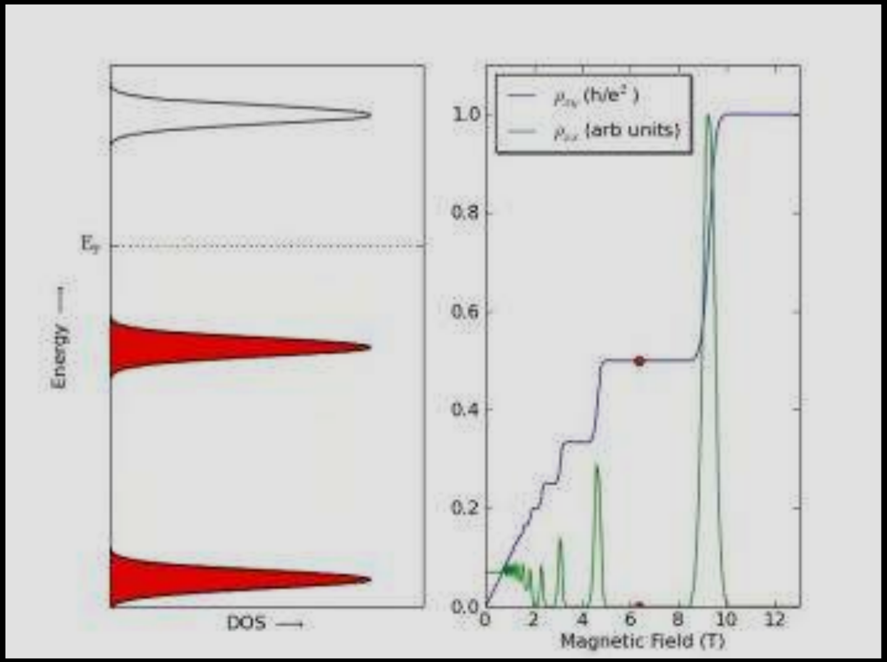
Robert Laughlin
Stanford

- more than 60 plateaux in LLL
- particle-hole asymmetry of plateaux (see 2/3 and 1/3)
- absence of even denominators (few exceptions as 5/2)
- average linearity
- Rxx small where Rxy flat (energy gap)

IQHE: Disorder

broadens LL into bands
of localized states and a core of
conducting states

Increase B
= increase degeneracy of LL
= Fermi Energy (fixed density)
crosses localized states (plateaux)
and LL cores (jump of Rxy)



FQHE: Coulomb interaction

quasiparticles that undergo IQHE

(Laughlin, Halperin, Haldane, Jain, Moore, Wen, ...)

THE GROUND STATES OF THE FQHE

$$\Psi(z_1 \dots z_n) = \prod_{i<j} (z_i - z_j)^m \exp(-\tfrac{1}{4\ell^2} \sum_k |z_k|^2)$$

Laughlin's ansatz for the GS at filling fraction 1/m is surprisingly good!

This and other G.S. (Moore, Read, Rezayi, Jain …) are:

Vandermonde x JACK polynomials

They are eigenstates of the Calogero-Sutherland Hamiltonian in 1D (Haldane & Bernevig, 2008):

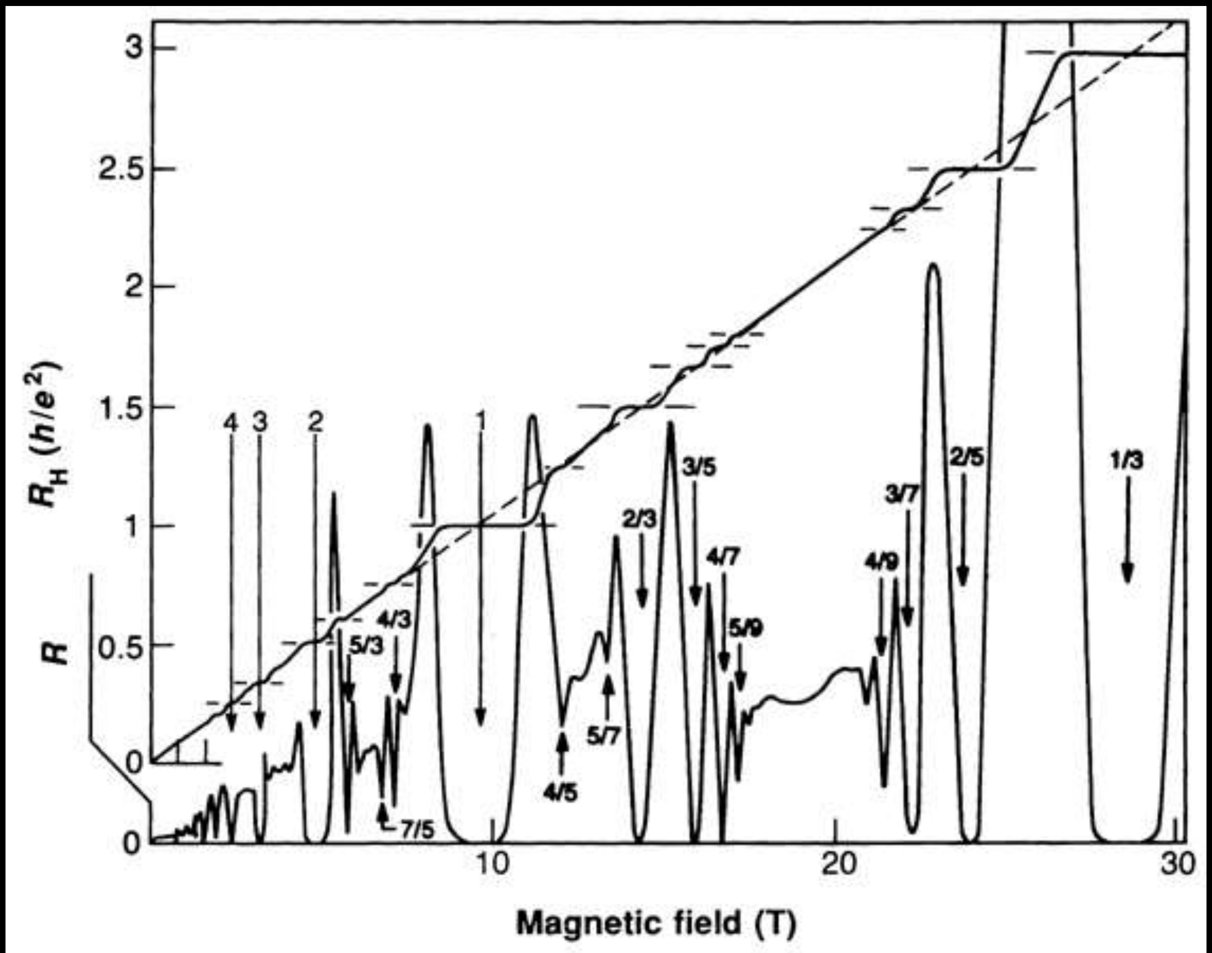
$$H_{CS}(\beta) = -\sum_{k=1}^{N} \frac{\partial^2}{\partial \theta_i^2} + \tfrac{1}{2}\beta(\beta-1) \sum_{j<k} \frac{1}{\sin^2 \frac{1}{2}(\theta_j - \theta_k)}$$

SUMMARY

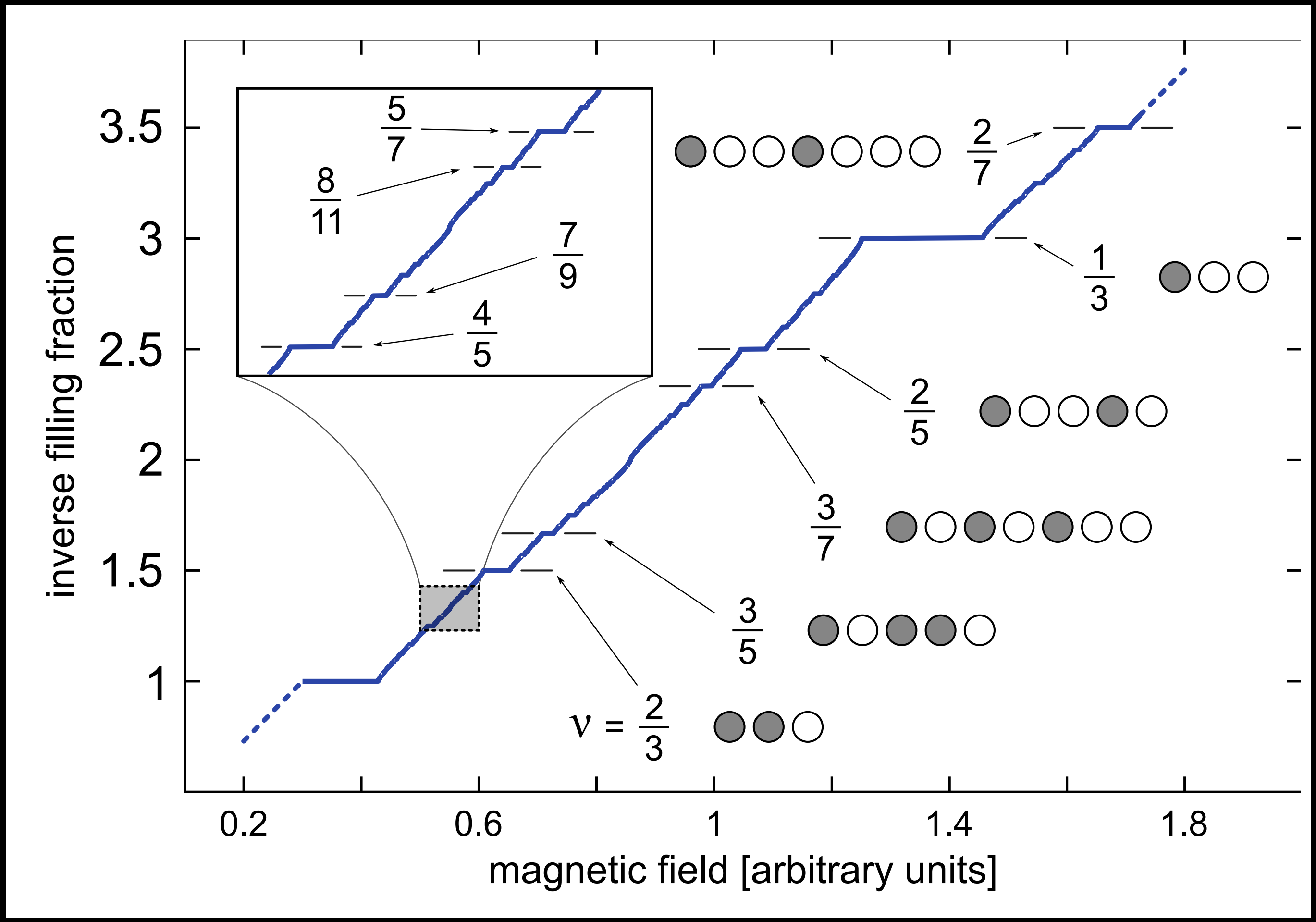
1) We map the Quantum Hall Hamiltonian restricted to the LL level in thin torus limit (Lx << magn. length << Ly) to a 1D long-range lattice model exactly solved by Hubbard;

2) going back, we qualitatively reproduce the experimental diagram of FQH

FQHE - EXPERIMENT



LATTICE CRYSTAL



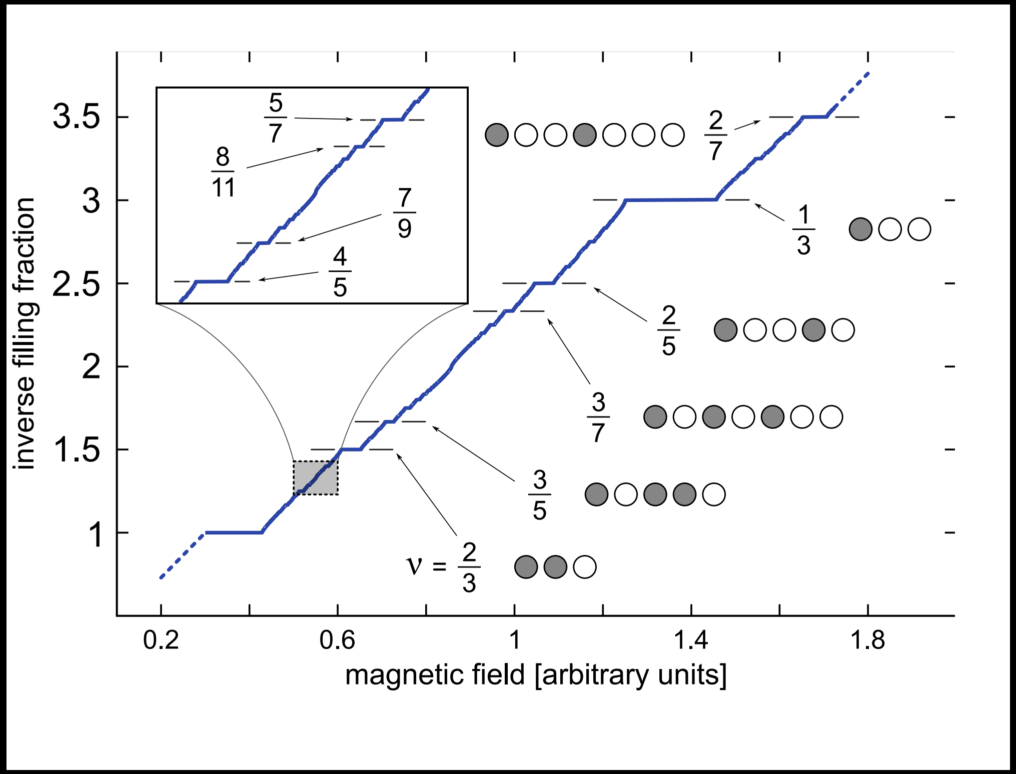
SUMMARY

3) We give in Fock space the OPERATOR

thin-torus state → Vandermonde x Jack Polynomial

ex: $\psi(1/3) = U(1/3)\,|\,100010001000...\}$

A "ground state" $\psi(p/q)$ (eigenstate of the Calogero-Sutherland Hamiltonian) is a superposition of number States |n1, n2, n3, ...} (Slater determinants) obtained by multiple squeezings of a single ROOT state



Interacting electrons in the lowest LL

$$\psi_s(\mathbf{r}) = \frac{1}{\sqrt{L_y}} \frac{1}{\pi^{\frac{1}{4}}\sqrt{\ell}} \exp\left[-\frac{1}{2}\left(\frac{x}{\ell} - \frac{2\pi\ell}{L_y} s\right)^2 - i\frac{2\pi}{L_y} sy \right], \quad 0 \le s \le N_s - 1$$

(Jacobi theta functions in torus geometry)

$H = e(B)N + \frac{1}{2} \Sigma <12|v|34> C_1 C_2 D_4 D_3$

$$\langle s_1, s_2 | v | s_3, s_4 \rangle = \frac{e^2}{L_y} \delta_{s_1+s_2, s_3+s_4} e^{-\frac{2\pi^2\ell^2}{L_y^2}(s_3-s_1)^2} \int_{-\infty}^{\infty} dq \frac{e^{-\frac{\ell^2}{2}q^2 + iq\frac{2\pi\ell^2}{L_y}(s_3-s_2)}}{\sqrt{q^2 + \frac{4\pi^2}{L_y^2}(s_3-s_1)^2}}$$

(Tao and Thouless, torus geometry)

a function of two parameters.
H can be diagonalized in the thin torus limit,
where $s_3 - s_2 = 0$ (Bergholtz-Karlhede, 2008)

THIN TORUS LIMIT Lx << ell << Ly

$$V(s_{13}) = \frac{e^2}{L_y} e^{-\frac{\pi^2 \ell^2}{L_y^2}(s_3 - s_1)^2} K_0\left(\frac{\pi^2 \ell^2}{L_y^2}(s_3 - s_1)^2\right)$$

From periodic lattice s= 1 ... Ns to dual lattice, via DFT

$$\ell^2[H_{LLL} - \mu' N] = -\left(\frac{\mu'}{eB} - \frac{1}{m}\right) N + L_x \sum_{k,k'} \frac{e^2}{|k-k'|} n_k n_{k'}$$

k=1, ... , Ns >> 1 (degeneracy of LLL)

Lattice Hamiltonian with long-range interaction and chemical potential μ(B).

The exact G.S. was obtained by Hubbard (1978)

PHYSICAL REVIEW B VOLUME 17, NUMBER 2 15 JANUARY 1978

Generalized Wigner lattices in one dimension and some applications to tetracyanoquinodimethane (TCNQ) salts

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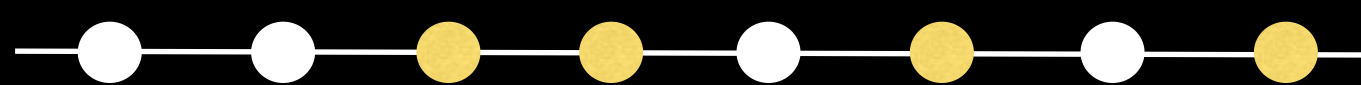
(Received 7 September 1977)

Estimates show that both the on-site and the near-neighbor electrostatic interactions in tetracyanoquinodimethane chains may be much greater than the bandwidth. A method of determining the exact ground state when the interactions are dominant is described; the electrons are found to have a periodic arrangement which may be regarded as a generalization of the classical Wigner lattice. It is shown how the optical spectra may be interpreted in terms of such a configuration; also that such arrangements may give rise to lattice distortions manifested as satellites in the x-ray diffraction pattern.



$$\boxed{E[n_1, n_2 \ldots] = \sum_{a,b} V(|b-a|) n_a n_b - \mu \sum_a n_a}$$

Find the occupation numbers n = 0,1
for minimum of E, with given density

A universal answer (independent of V) if
1) V(m) > 0 and decreases to zero
2) V(m+1) + V(m-1) > 2 V(m)

A hierarchy of ground states by continued fraction expansion of p/q

TABLE II. Generalized-Wigner-lattice configurations.

	Density	Period		Configuration
(a)	$\frac{1}{3}$	3	3	100100100...
(b)	$\frac{2}{5}$	5	32	100101001010010...
(c)	$\frac{3}{7}$	7	$2^2 3$	10101001010100...
(d)	$\frac{3}{8}$	8	$3^2 2$	1001001010010010...
(e)	$\frac{10}{23}$	23	$(2^2 3)^2 2^3 3$	10101001010100101010100...
(f)	$\frac{3}{5}$	5	12^2	1101011010...
(g)	$\frac{3}{4}$	4	$1^2 2$	111011101110...
(h)	$\frac{4}{7}$	7	12^3	11010101101010...
(i)	$\frac{1}{2}$	2	2	1010101010...
(j)	$\frac{1}{2}$	4	13	110011001100...

particles wish to stay as far as possible
but the lattice constrains their positions

VOLUME 49, NUMBER 4 PHYSICAL REVIEW LETTERS 26 JULY 1982

One-Dimensional Ising Model and the Complete Devil's Staircase

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and

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(Received 18 March 1982)

It is shown rigorously that the one-dimensional Ising model with long-range antiferromagnetic interactions exhibits a complete devil's staircase.

PACS numbers: 05.50.+q, 75.10.Hk

$$\boxed{\Delta\mu = 2q \sum_{k=1}^{\infty} k[V(kq+1) + V(kq-1) - 2V(kq)]}$$

The devil's staircase

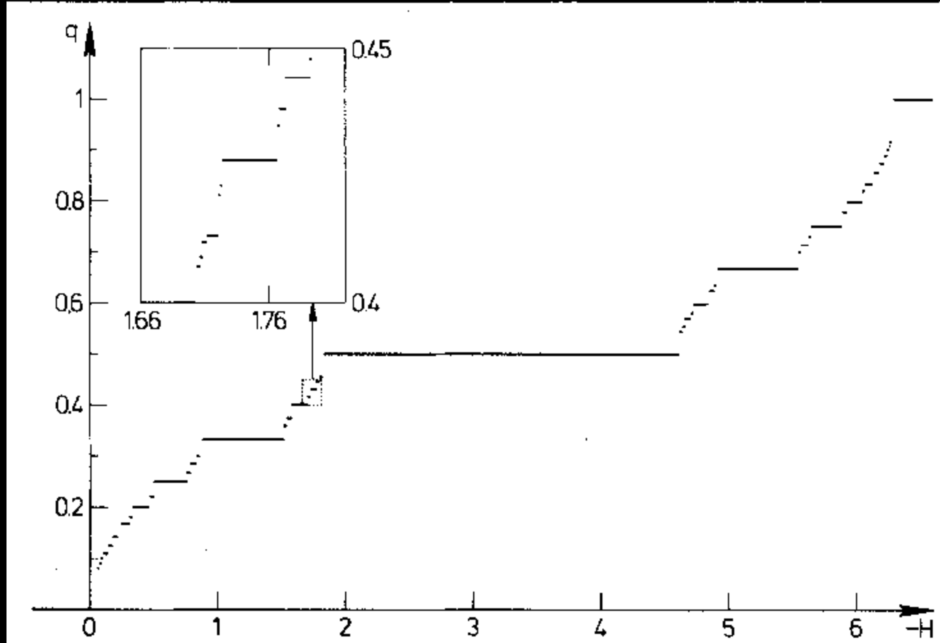
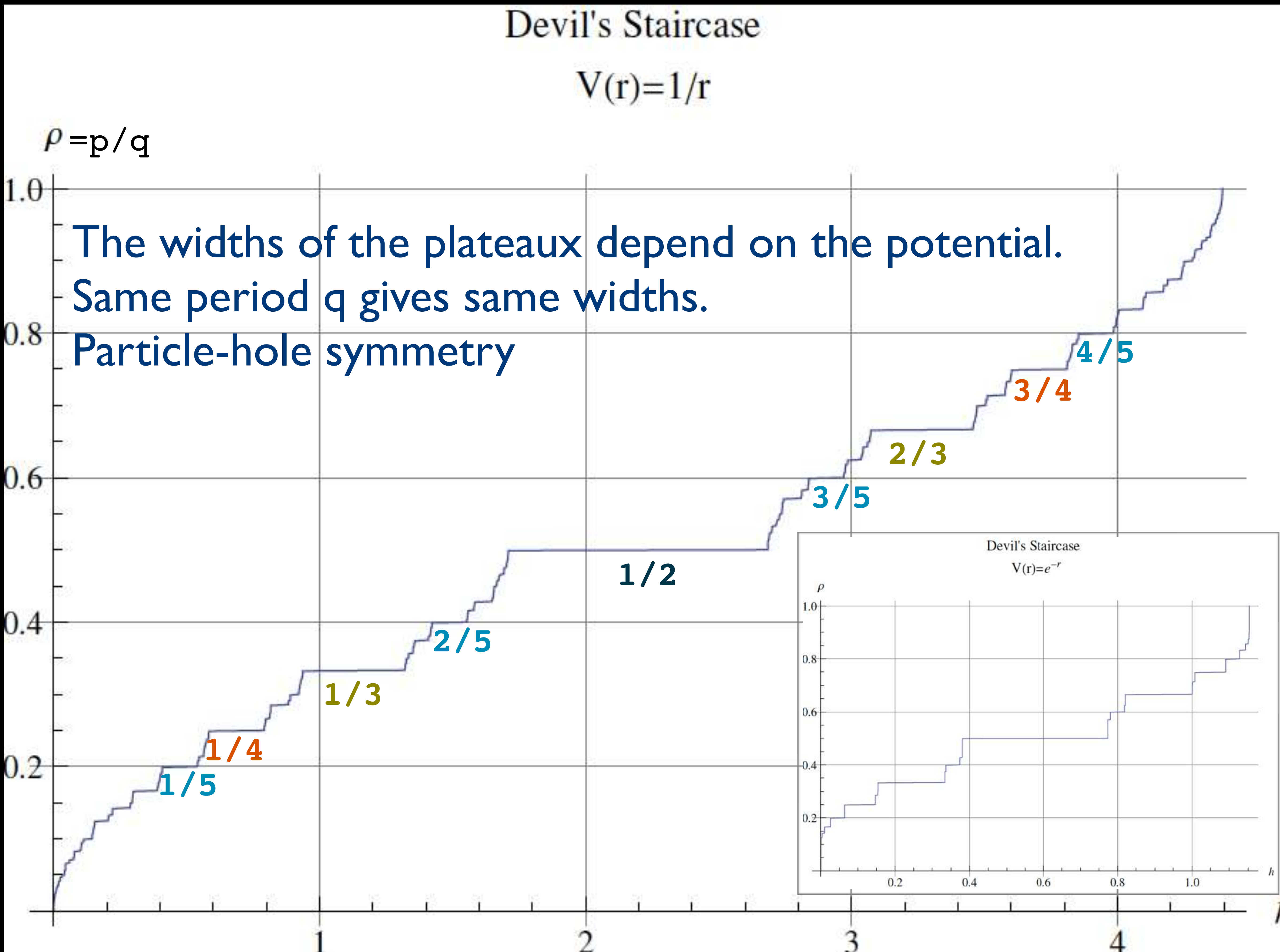


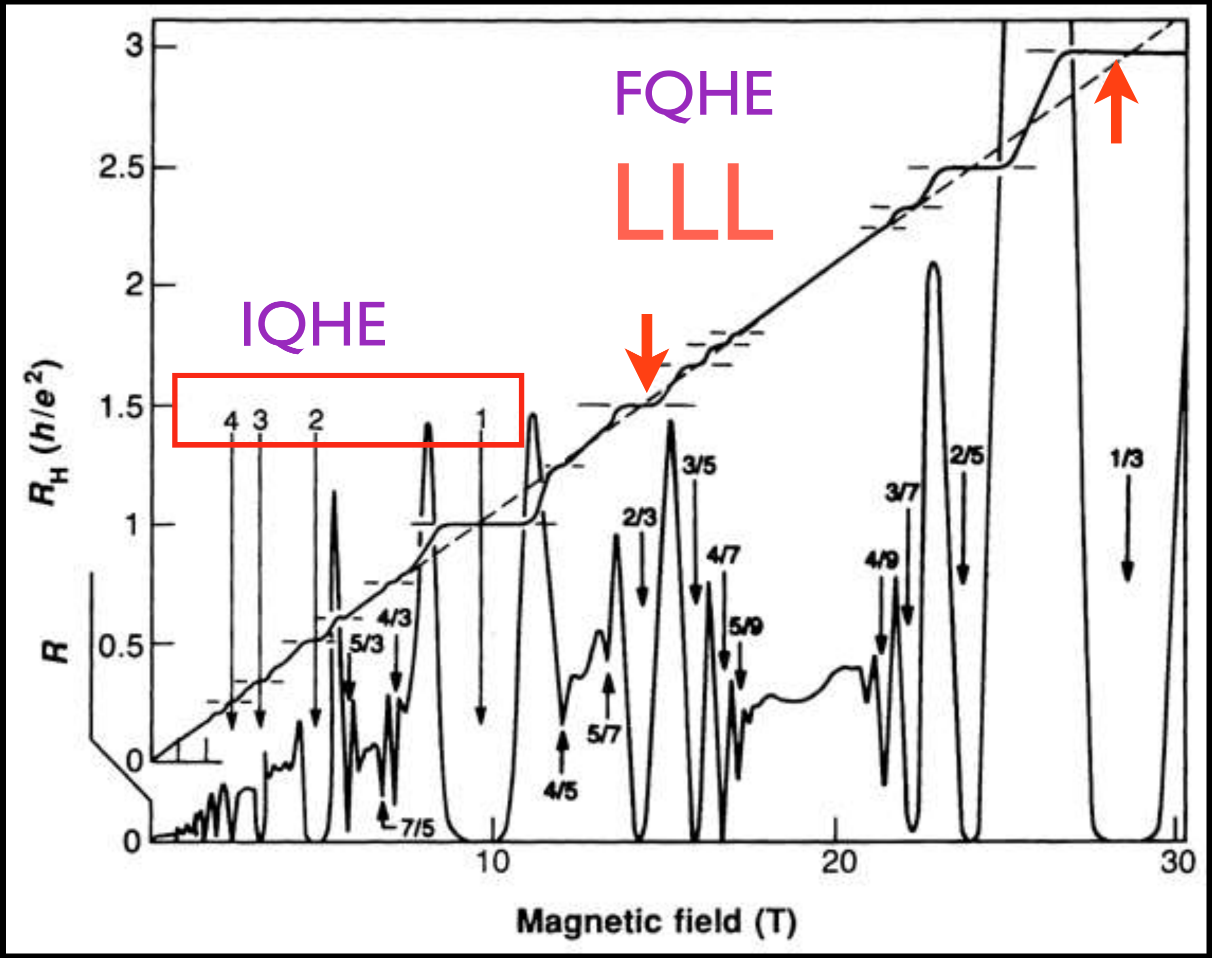
FIG. 2. The devil's staircase. The ratio of up spins over down spins q is plotted vs the applied field H for an interaction $J(i) = i^{-2}$. Inset: The area in the square magnified 10 times.

(see also Burkov, Sinai, 1983)

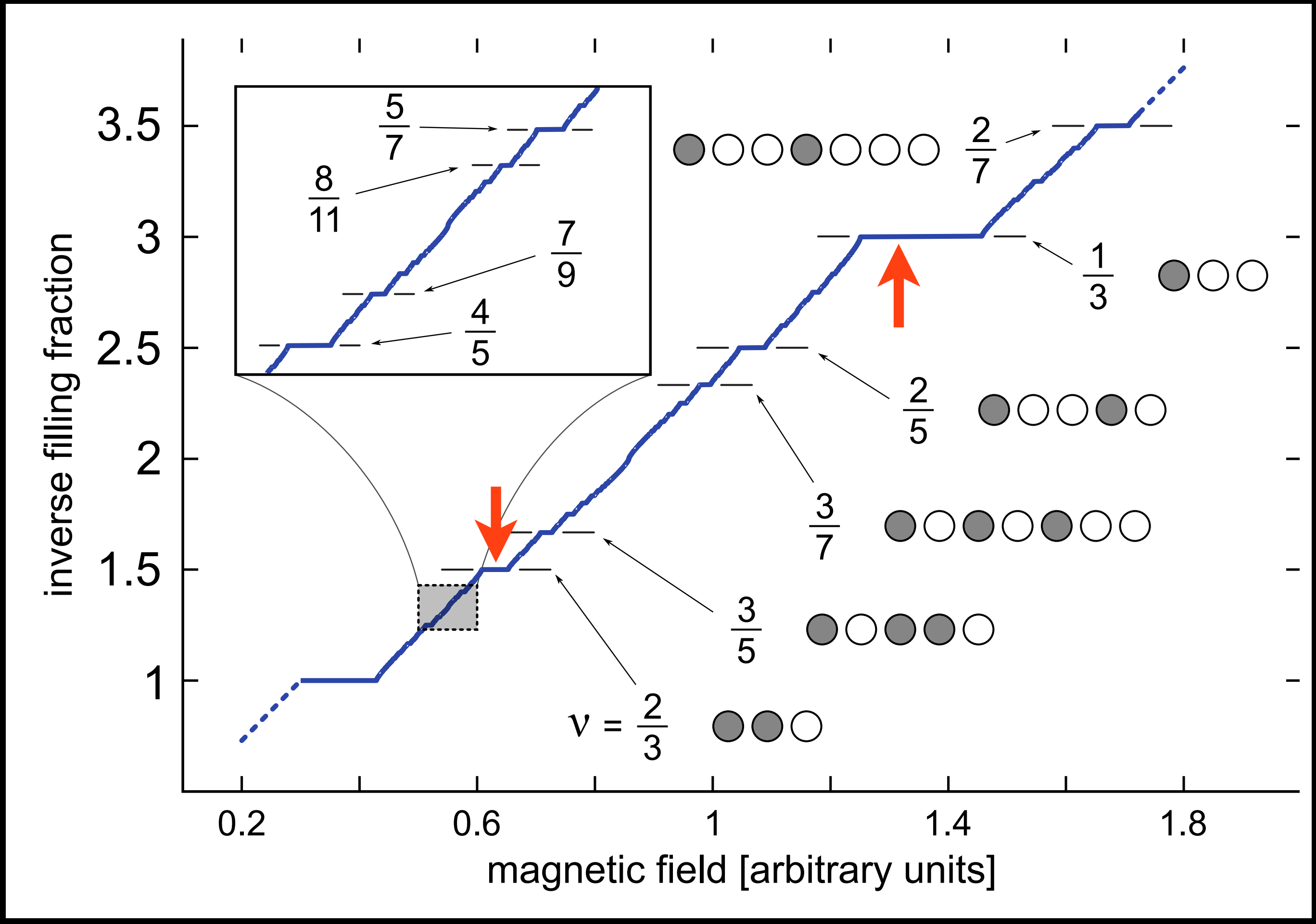

Devil's Staircase
V(r)=1/r
ρ =p/q
The widths of the plateaux depend on the potential.
Same period q gives same widths.
Particle-hole symmetry
1/5
1/4
1/3
2/5
1/2
3/5
2/3
3/4
4/5
h
Devil's Staircase
V(r)=e^{-r}

Surprisingly, nobody ever translated
the plot density - μ of the lattice gas
to a plot inverse density (filling fr) - B for the thin torus FQHE

QHE - experiment



Lattice crystal



- while rescaling μ~ 1/B, plateaux with same q become narrower for higher p in accordance with experiments.
- absence of even denominators (few exceptions, as 5/2)
- average linearity

Absence of even denominators
(magnetic symmetries & Fermi statistics)

PRL **105**, 026802 (2010) PHYSICAL REVIEW LETTERS week ending 9 JULY 2010

S-Duality Constraints on 1D Patterns Associated with Fractional Quantum Hall States

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(Received 23 February 2010; published 6 July 2010)

Using the modular invariance of the torus, constraints on the 1D patterns are derived that are associated with various fractional quantum Hall ground states, e.g., through the thin torus limit. In the simplest case, these constraints enforce the well-known odd-denominator rule, which is seen to be a necessary property of all 1D patterns associated to quantum Hall states with minimum torus degeneracy. However, the same constraints also have implications for the non-Abelian states possible within this framework. In simple cases, including the $\nu = 1$ Moore-Read state and the $\nu = 3/2$ level 3 Read-Rezayi state, the filling factor and the torus degeneracy uniquely specify the possible patterns, and thus all physical properties that are encoded in them. It is also shown that some states, such as the "strong p-wave pairing state," cannot in principle be described through 1D patterns.

Calogero Sutherland & Laplace Beltrami in Fock space

$$H_{LB} = \Delta^{-\beta} H_{CS} \Delta^{\beta}$$

$$H_{CS}(\beta) = -\sum_{k=1}^{N} \frac{\partial^2}{\partial\theta_i^2} + \tfrac{1}{2}\beta(\beta-1)\sum_{j<k} \frac{1}{\sin^2 \frac{1}{2}(\theta_j - \theta_k)}$$

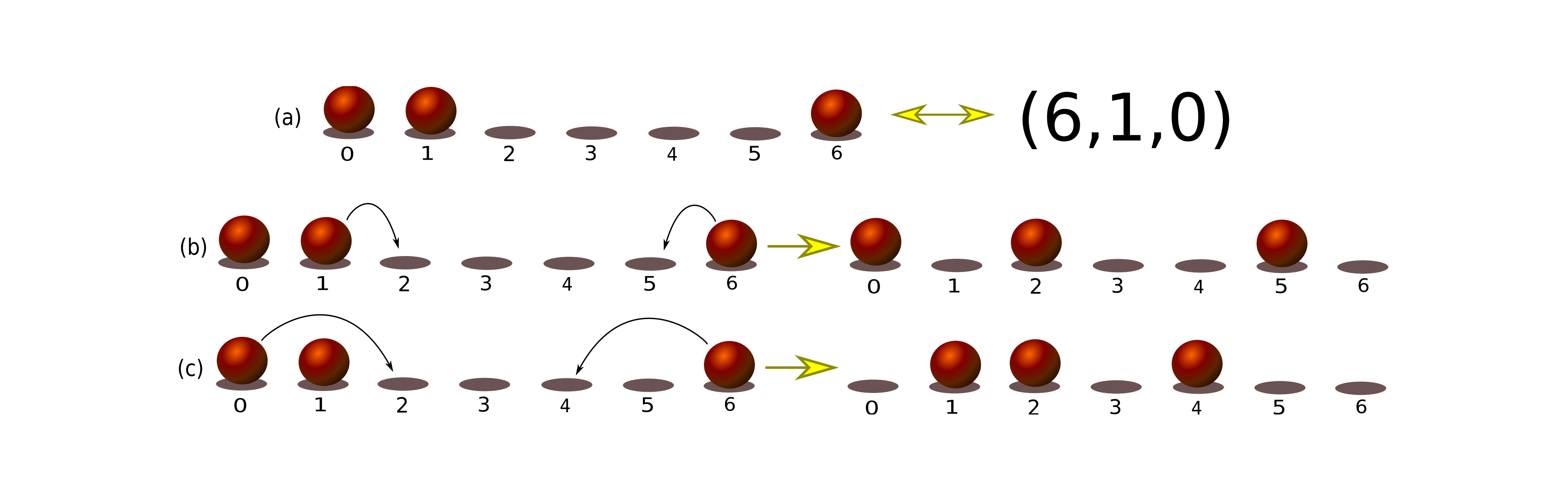
z=exp (iθ)

$$\boxed{H_{LB} = \sum_{k=1}^{N} (z_k\partial_k)^2 + \frac{\beta}{2}\sum_{j\neq k} \frac{z_k + z_j}{z_k - z_j}(z_k\partial_k - z_j\partial_j)}$$

$$\boxed{H_{LB} = \sum_{r=0}^{\infty} r^2 n_r + \beta \sum_{u=1}^{\infty}\sum_{m=0}^{u-1}\sum_{k=1}^{u-m} (u-m) b^{\dagger}_{u-k} b^{\dagger}_{m+k} b_m b_u}$$

SQUEEZING !

Squeezing of Slater states (a 2-particle operation)



Conservation of angular momentum

$$\{C_i, C_j\} = 0 \quad \{D_i, D_j\} = 0 \quad \{C_i, D_j\} = \boldsymbol{\delta}_{ij}$$

$$\text{(b) } |0110010\} \sim C_2 C_5 D_1 D_6 \, |1100001\}$$

IN FOCK SPACE: $\mathbf{H}_{LB} = \mathbf{D} + \mathbf{S}$.

D contains number operators

S is a sum of squeezing operators

PROPOSITION:

if : $\mathbf{D}\ |\text{Slater}\rangle = \mathbf{E}\ |\text{Slater}\rangle$

E non-degenerate

then : $\mathbf{H}_{LB}\, \psi = \mathbf{E}\, \psi$

$$\psi = [1 - (E - \mathbf{D})^{-1} \mathbf{S}\,]^{-1}\, |\text{Slater}\rangle$$

A Fock space formulation of the theory of Jack polynomials

the question remains:

what is JACK doing on a devil's staircase ?