Quantum Control for Advanced Quantum Metrology

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Outline

- Quantum metrology
  - ✓ standard quantum limit & Heisenberg limit
  - ✓ an example of a quantum resource: squeezing
- Time-continuous measurements to generate quantum squeezing
  - ✓ an example in an optomechanical system (a levitating nanosphere)
- Time-continuous measurements for noisy quantum magnetometry
  - ✓ no-go theorems for noisy quantum metrology
  - ✓ can we recover the Heisenberg limit ?
  - ✓ can we do better than this ?
- Outlooks

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Quantum estimation theory

$p(x|\theta) = \mathrm{Tr}[\varrho_\theta \Pi_x]$ conditional probability

Π_x projection on the corresponding eigenstate

Bound on Estimation Precision (classical and quantum Cramér-Rao bounds)

$$\mathrm{Var}_{\hat{\theta}}(\theta) \geq \frac{1}{M\mathcal{F}[p(x|\theta)]}$$

Paris, IJQI **7**, 125 (2009)

number of measurements: M

Fisher Information: $\mathcal{F}[p(x|\theta)] = \int dx\, p(x|\theta)(\partial_\theta \log p(x|\theta))^2$ achievable via optimization over classical estimators at fixed measurement

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$$\mathrm{Var}_{\hat{\theta}}(\theta) \geq \frac{1}{M\mathcal{F}[p(x|\theta)]} \geq \frac{1}{M\mathcal{Q}[\varrho_\theta]}$$

the Quantum Fisher Information is the correct quantity to assess a quantum estimation problem

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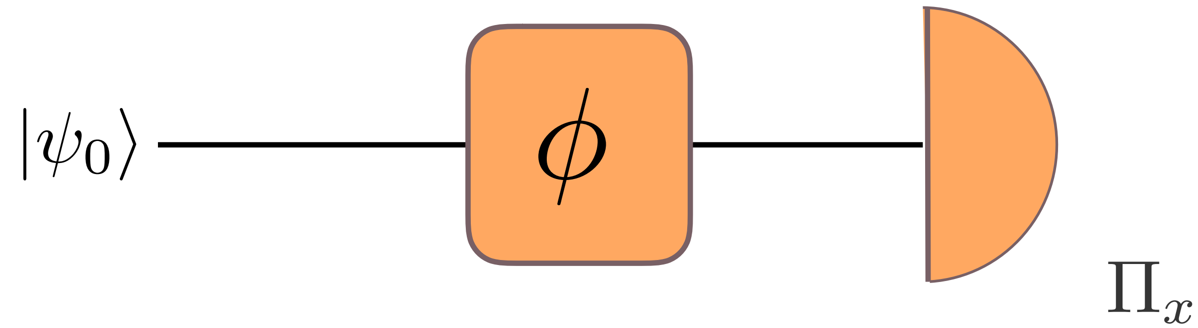
quantum Fisher Information: $\mathcal{Q}[\varrho_\theta] = \mathrm{Tr}[\varrho_\theta L_\theta^2]$

with $2\partial_\theta \varrho_\theta = L_\theta \varrho_\theta + \varrho_\theta L_\theta$

ultimate bound achievable via optimization over all the possible quantum measurements

Quantum metrology

$$|\psi_\phi\rangle = \exp\{-i\phi\hat{H}\}|\psi_0\rangle$$



$$\mathcal{Q}_\phi[\psi_\phi] = 4\langle\Delta\hat{H}^2\rangle_{\psi_0}$$

Let us now assume that we have at disposal an initial state with "**N resources**"
(e.g. N = number of probes, qubits, photons, total spin...)

$|\psi_0^{(c)}\rangle$ CLASSICAL STATE (e.g. coherent states)

$\mathcal{Q}_\phi \sim N$ **Standard Quantum Limit (SQL)**

linear scaling with the number of resources N

Giovannetti, Maccone & Lloyd, Nat. Phys. **5** (2011)

Quantum metrology

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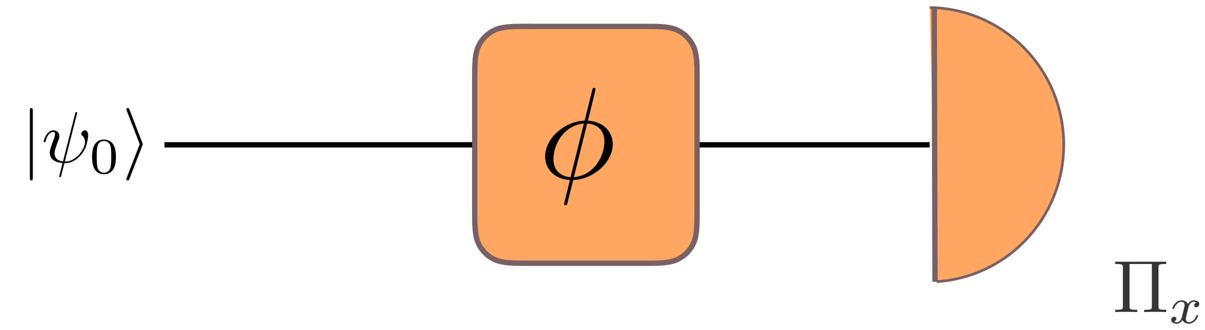

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Let us now assume that we have at disposal an initial state with "**N resources**"
(e.g. N = number of probes, qubits, photons, total spin...)

$|\psi_0^{(nc)}\rangle$ "NON-CLASSICAL" STATE (i.e. with quantum resources such as squeezing and entanglement)

$$\mathcal{Q}_\phi \sim N^2$$

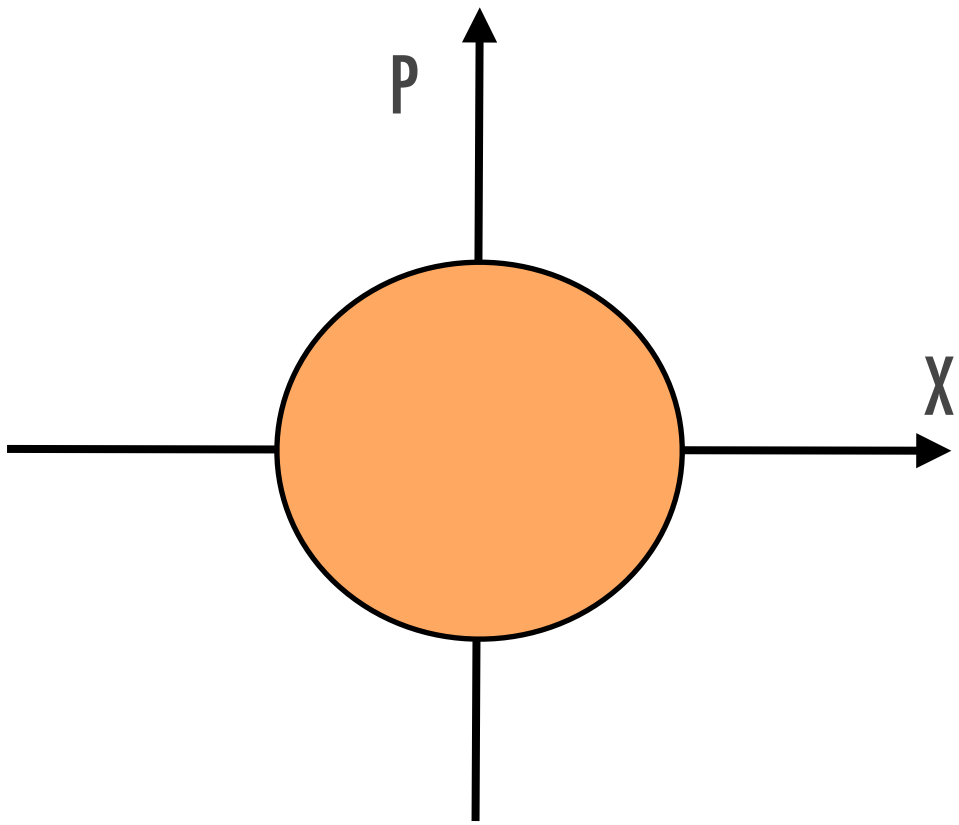
Heisenberg Limit (HL)

quadratic scaling with the number of resources **N**

Giovannetti, Maccone & Lloyd, Nat. Phys. **5** (2011)

What is quantum squeezing? (and why is useful?)

Phase-space picture of a quantum harmonic oscillator [X,P] = i



CLASSICAL STATES

THERMAL STATE $\Delta x^2 = \Delta p^2 > 1$

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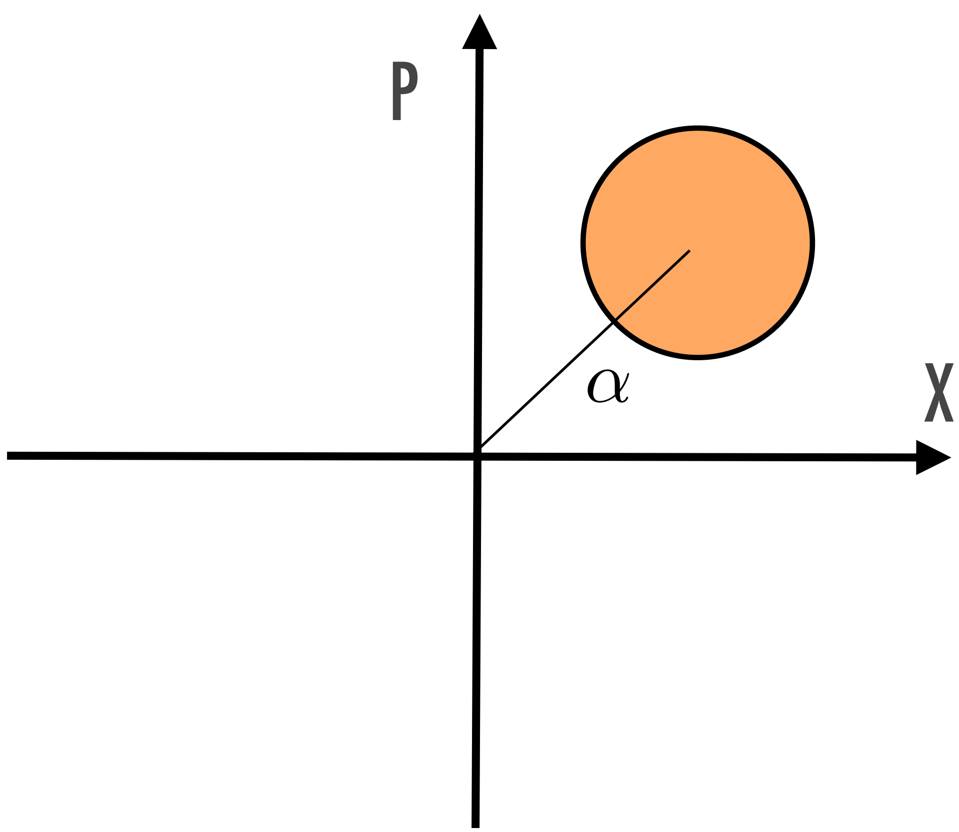

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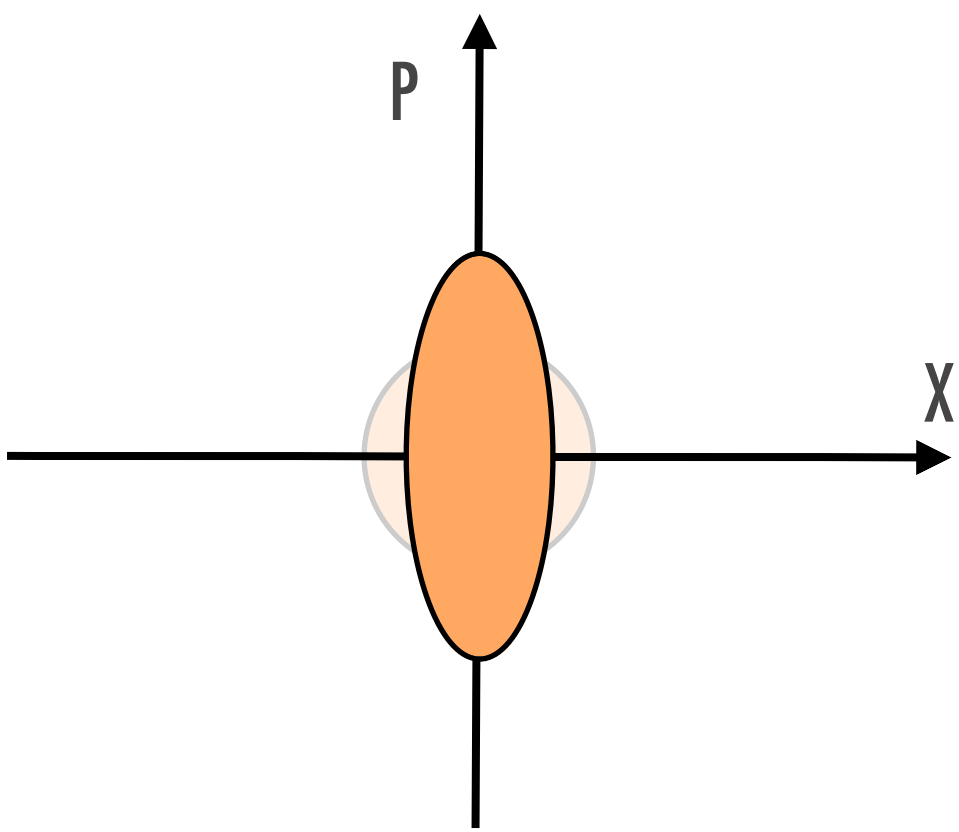

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SQUEEZED STATE $\Delta x^2 < 1$ one quadrature with sub shot-noise fluctuations

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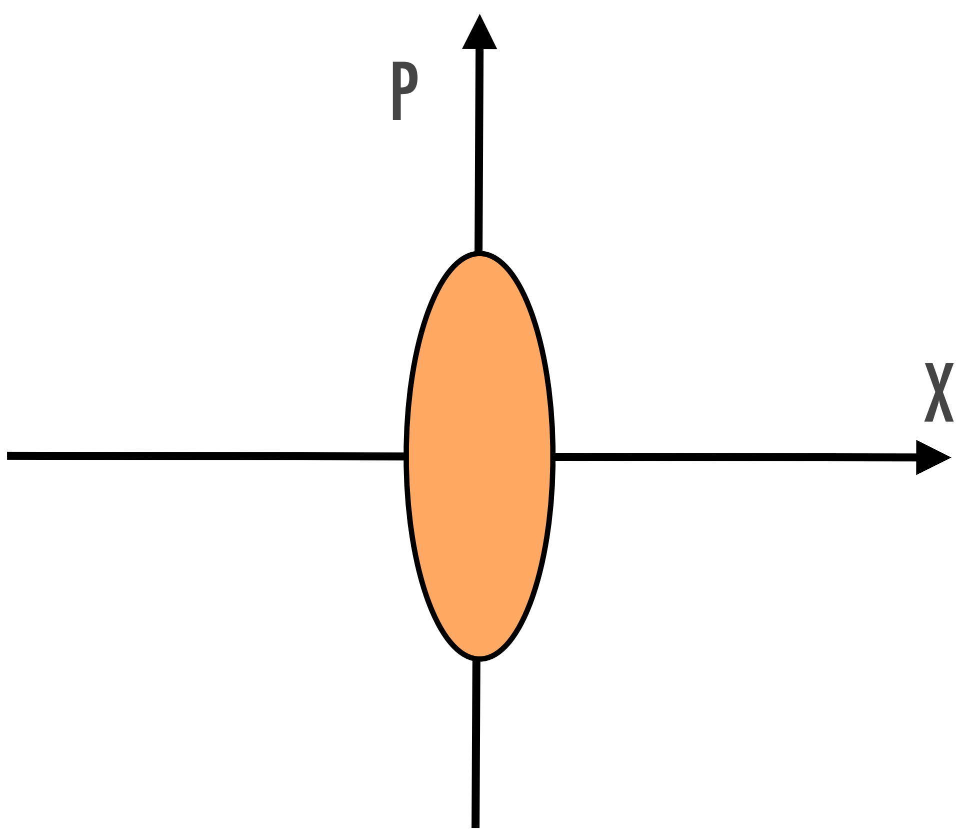

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WHY IS SQUEEZING USEFUL FOR METROLOGY ?

Sub-shot noise fluctuations allow to measure at the Heisenberg limit:

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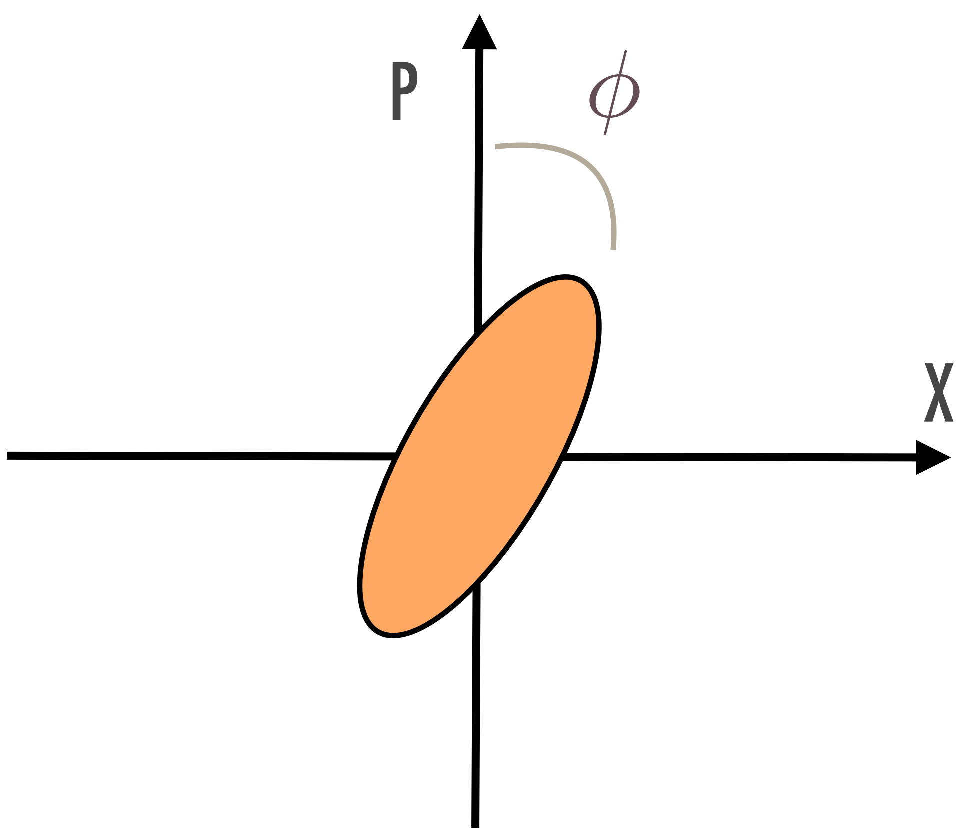

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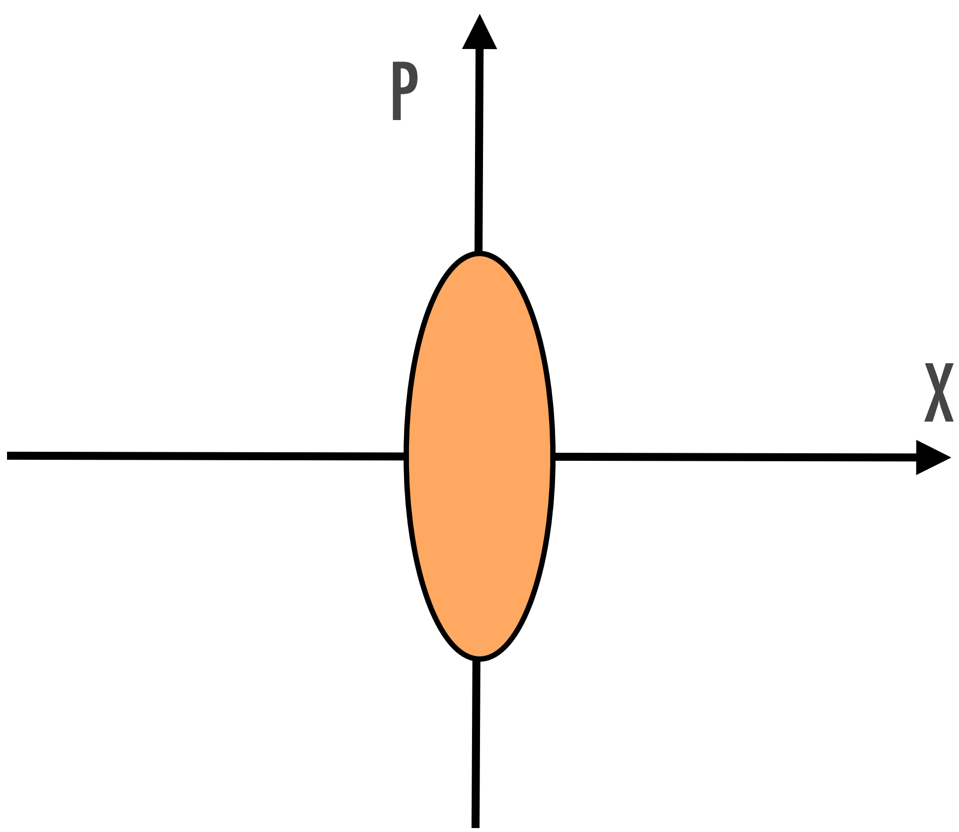

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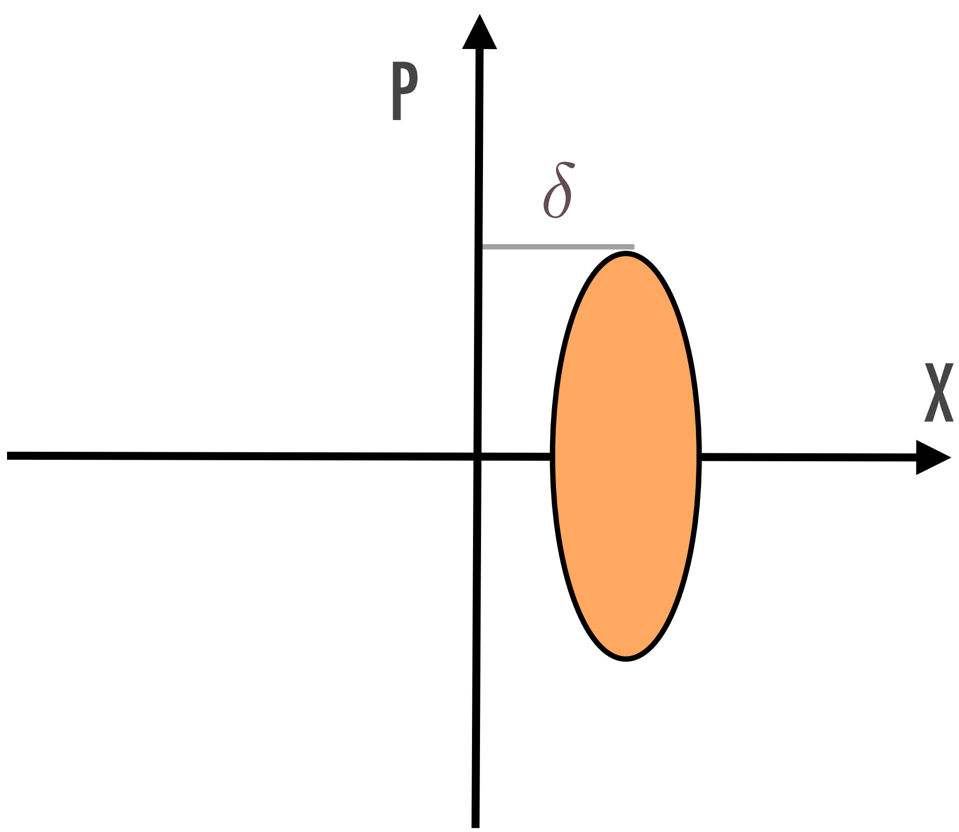

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- ✓ phase rotations in phase space — Monras, PRA (2006) — Genoni et al., PRL (2011)
- ✓ displacements — Genoni et al., PRA (2013)

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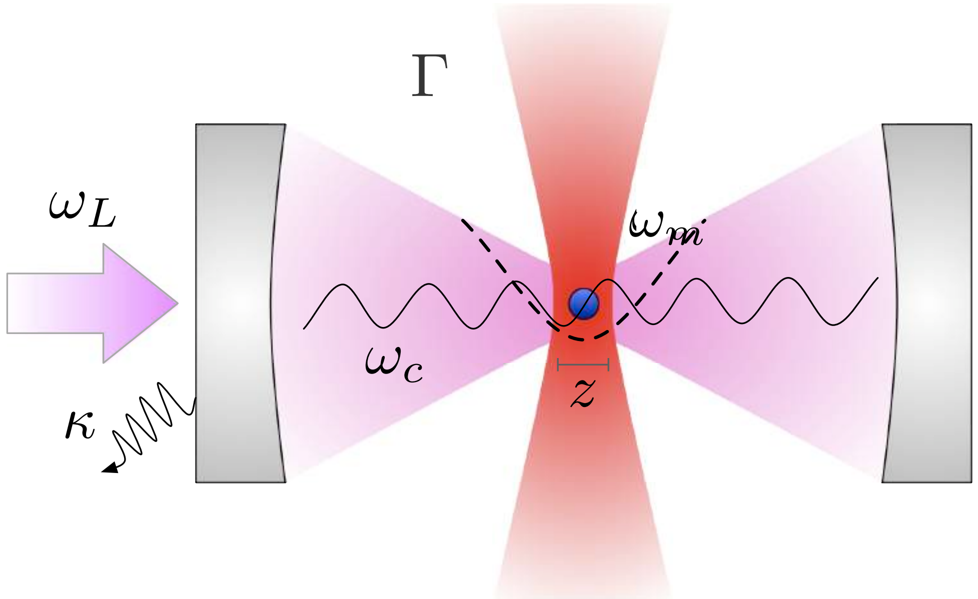
Quantum optomechanics with a levitated nanosphere



Hamiltonian (linearized assuming cavity strongly driven)

$$H = \omega_m b^\dagger b - \Delta a^\dagger a + g(a + a^\dagger)(b + b^\dagger)$$

$\Delta = \omega_L - \omega_c$ detuning from cavity resonance

Romero-Isart et al., PRA **83** (2011)

- ✓ promising platforms for ultra-sensitive detectors thanks to the ability of mechanical systems to respond to optical, electrical or magnetic forces.
- ✓ ideal playground to test fundamental physics: "quantum-to-classical transition", non-linear corrections to Schroedinger equation, collapse models.

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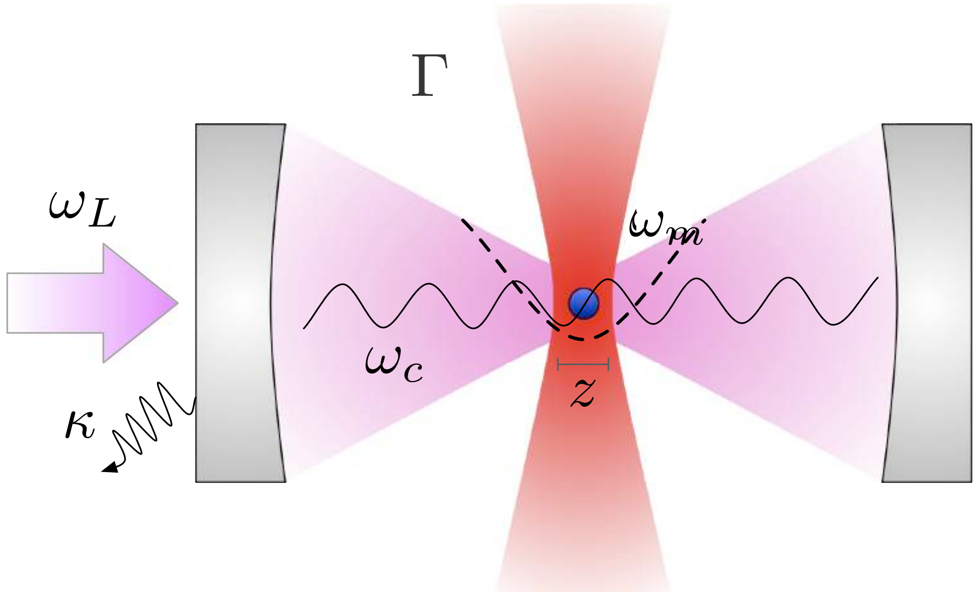

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Master Equation

$$\mathcal{D}[A]\varrho = A\varrho A^\dagger - \frac{1}{2}\{A^\dagger A, \varrho\}$$

$$\dot{\varrho} = i[\varrho, H] + \kappa\mathcal{D}[a]\varrho + \Gamma\mathcal{D}[b + b^\dagger]\varrho$$

cavity loss **recoil heating (light scattering)**

What can we achieve using side-band cooling ?

- ✓ stabilize the dynamics, only for red-detuning, to a thermal state with no less than 50 phonons
- ✓ basically impossible to observe any quantum squeezing (with current experimental setups)

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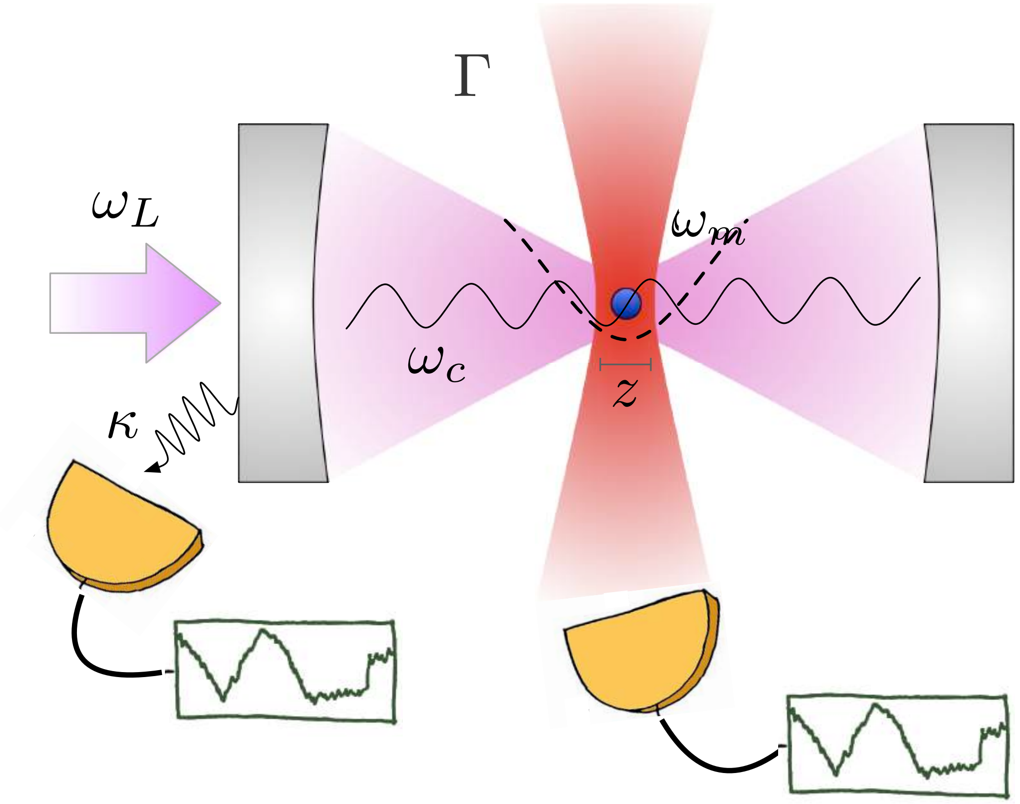

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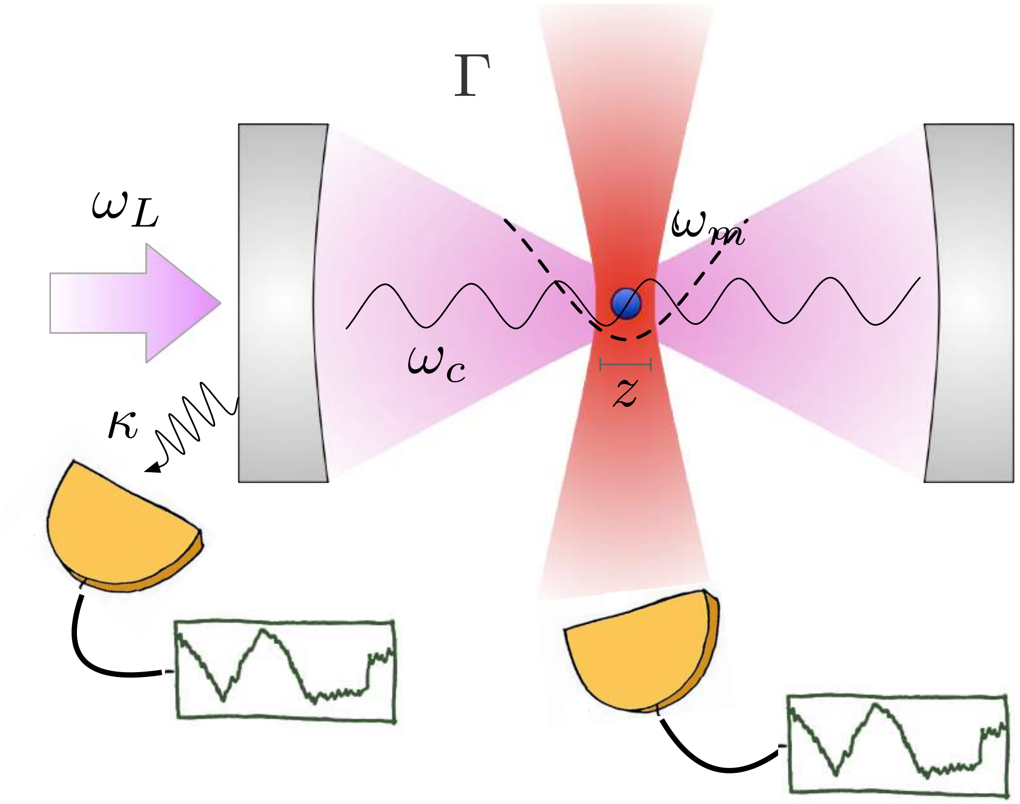
Assumption
It is possible to monitor (measure) the "output" of system on time-scales which are much shorter than the typical system's response time.

Wiseman & Milburn,
Quantum Measurement and Control
(2010)

Quantum optomechanics with a levitated nanosphere



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Stochastic Master Equation

$$d\varrho = i[\varrho, H]dt + \kappa\mathcal{D}[a]\varrho\, dt + \Gamma\mathcal{D}[b + b^\dagger]\varrho\, dt$$
$$+ \sqrt{\kappa\eta_1}\,\mathcal{H}[ae^{i\phi}]\varrho\, dw_1 + \sqrt{\Gamma\eta_2}\,\mathcal{H}[b + b^\dagger]\varrho\, dw_2$$

homodyne measurement of the cavity mode

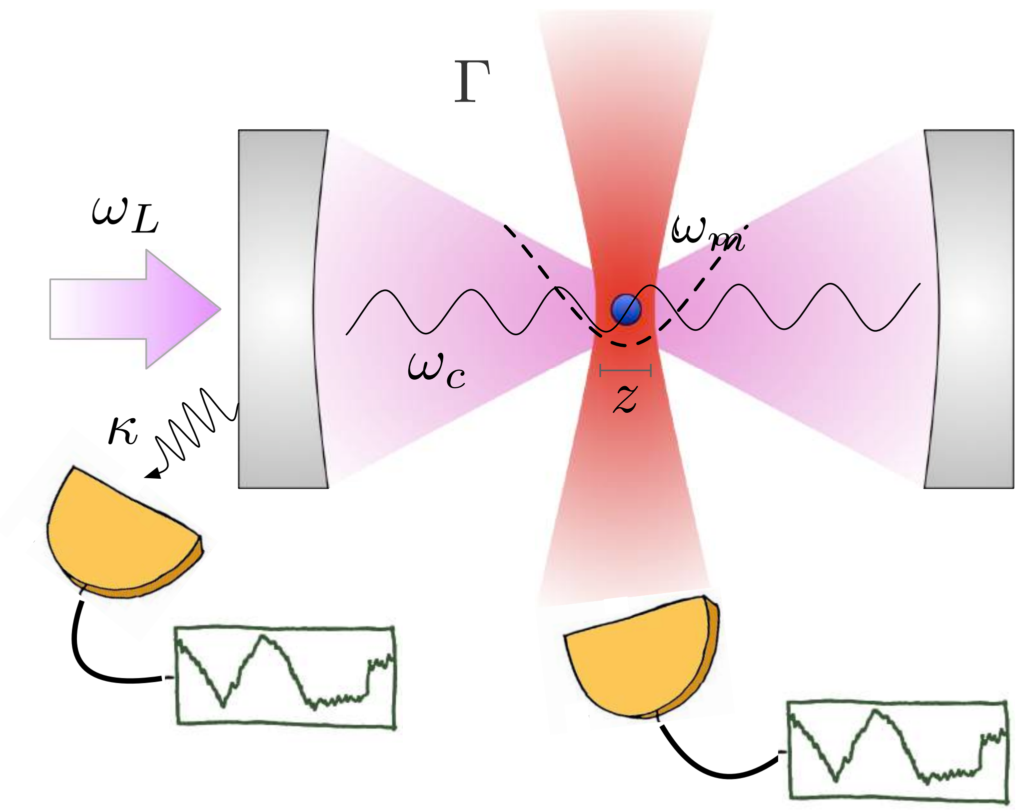
position measurement of the oscillator

$$\mathcal{H}[A]\varrho = A\varrho + \varrho A^\dagger - \langle A + A^\dagger\rangle\varrho$$

η_j measurement efficiencies

dw_j Wiener (stochastic) increments

Quantum optomechanics with a levitated nanosphere



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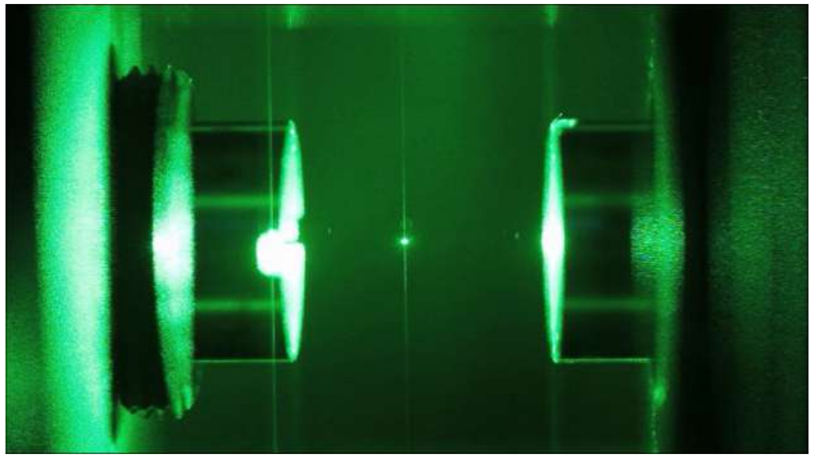
Simulations for UCL setup

Genoni et al., NJP **17** (2015)

- ✓ stabilize the dynamics for any value of the detuning
- ✓ possibility to cool (via also linear feedback) the oscillator down to less than 1 phonon (almost the ground state)
- ✓ possibility to observe quantum squeezing by improving the measurement efficiencies



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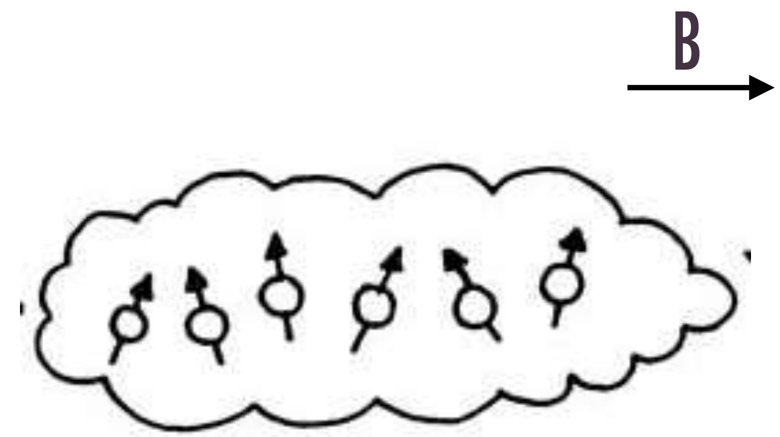
Quantum magnetometry (frequency estimation)

$$\hat{H} = \omega \hat{J}_y = \omega \sum_{k=1}^{2J} \hat{\sigma}_y^{(k)} \quad \text{where} \quad \omega = \gamma B$$

$$|\psi_\omega\rangle = \exp\{-i\omega t\, \hat{J}_y\}\, |\psi_0\rangle$$

$$\mathcal{Q}_\omega[\psi_\omega] = 4t^2 \langle \Delta \hat{J}_y^2 \rangle_{\psi_0}$$



N two-level atoms
J=N/2 spin

Wineland et al., PRA **46**, R6797 (1992)

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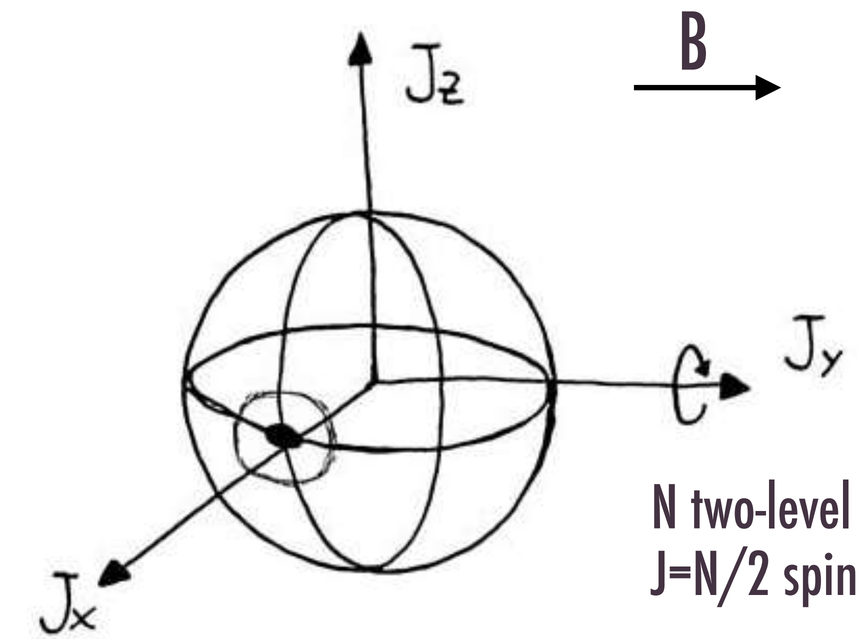

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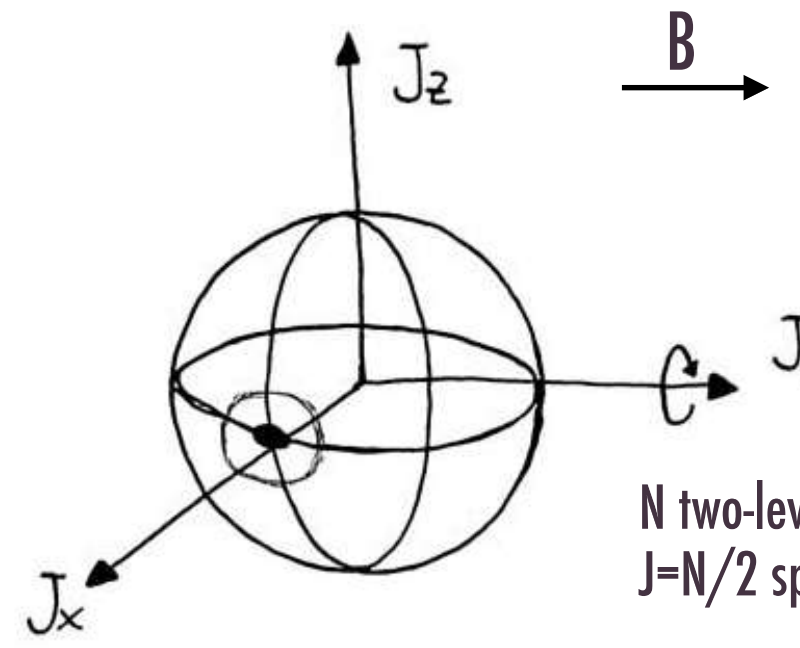

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$$|\psi_0\rangle = \bigotimes_{k=1}^{2J} |+\rangle_x = |J, J\rangle_x$$

SPIN COHERENT STATE

$$\mathcal{Q}_\omega \sim J$$

Standard Quantum Limit (SQL)

linear scaling with the total spin J

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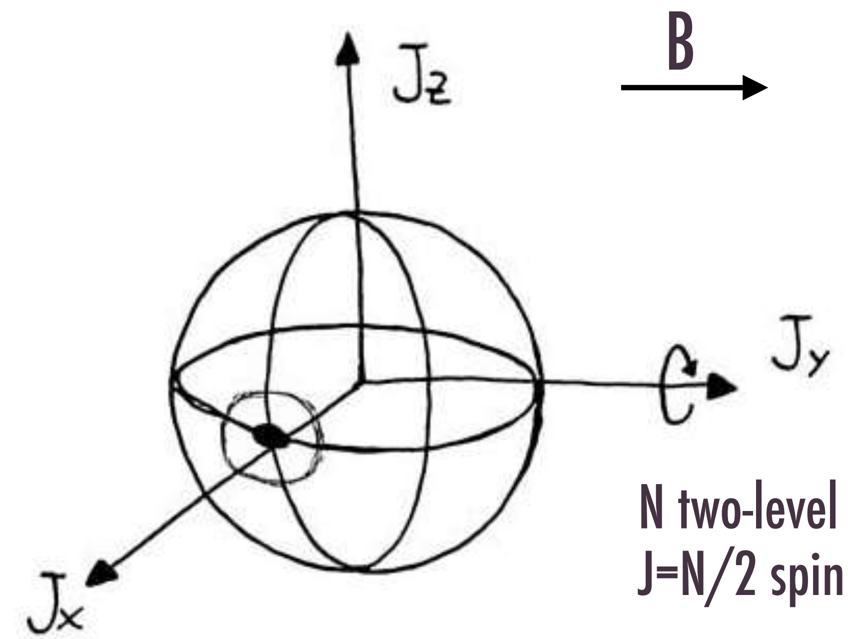

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Wineland et al., PRA **46**, R6797 (1992)

$$|\psi_0\rangle = (|J,J\rangle_x + |J,-J\rangle_x)/\sqrt{2}$$ GHZ STATE

$$\mathcal{Q}_\omega \sim J^2$$ **Heisenberg Limit (HL)**

quadratic scaling with the total spin **J**

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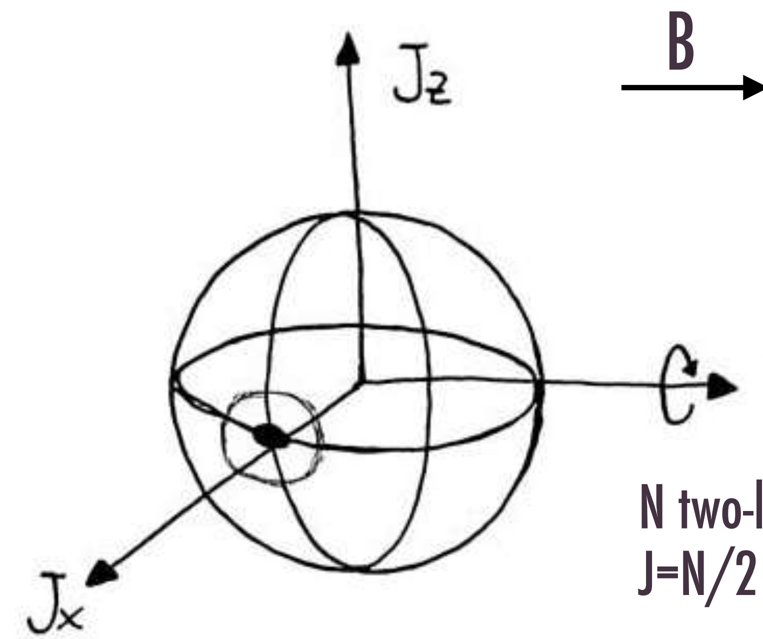

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Wineland et al., PRA **46**, R6797 (1992)

$$|\psi_0\rangle = |SST\rangle$$ SPIN-SQUEEZED STATE (with reduced variance in z-direction)

$$\mathcal{Q}_\omega \sim J^2$$ **Heisenberg Limit (HL)**

quadratic scaling with the total spin **J**

Noisy quantum magnetometry

what if we introduce some **noise** on the dynamics ?

NO-GO THEOREMS

for most types of noise acting independently on each probe particle, an **infinitesimally small** amount of **decoherence** limits any **quantum improvement** over the **SQL** to at most a **constant factor**

$$\mathcal{Q}_\omega \sim J$$

Standard Quantum Limit (SQL)

Huelga et al., PRL **79**, 3865 (1997)

Escher et al., Nat. Phys. **7**, 406 (2011)

Demkowicz-Dobrzanski et al., Nat. Comm **3**, 1063 (2012)

In our framework, this applies to different kind of noise

$$\frac{d\varrho}{dt} = -i\omega[\hat{J}_y, \varrho] + \kappa\mathcal{D}[\hat{J}_y]\varrho$$

pure dephasing

noise parallel to the generator

$$\frac{d\varrho}{dt} = -i\omega[\hat{J}_y, \varrho] + \kappa\mathcal{D}[\hat{J}_z]\varrho$$

transversal noise

noise perpendicular to the generator

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pure dephasing

Is there any way to "go around" these no-go theorems and observe a non-classical scaling?

$$\frac{d\varrho}{dt} =$$

noise perpendicular to the generator

Noisy quantum magnetometry

How to circumvent these no-go theorems?

- Quantum error-correction methods

Kessler et al., PRL (2014)

Dur et al., PRL (2014)

Arrad et al., PRL (2014)

- time-inhomogeneous (non-semigroup, "non-Markovian") dephasing noise

Matsuzaki et al., PRA (2011)

Chin et al., PRL (2012)

- noise with a particular geometry

Chaves et al., PRL (2013)

Brask et al., PRX (2015)

Smirne et al., PRL (2016)

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transversal noise

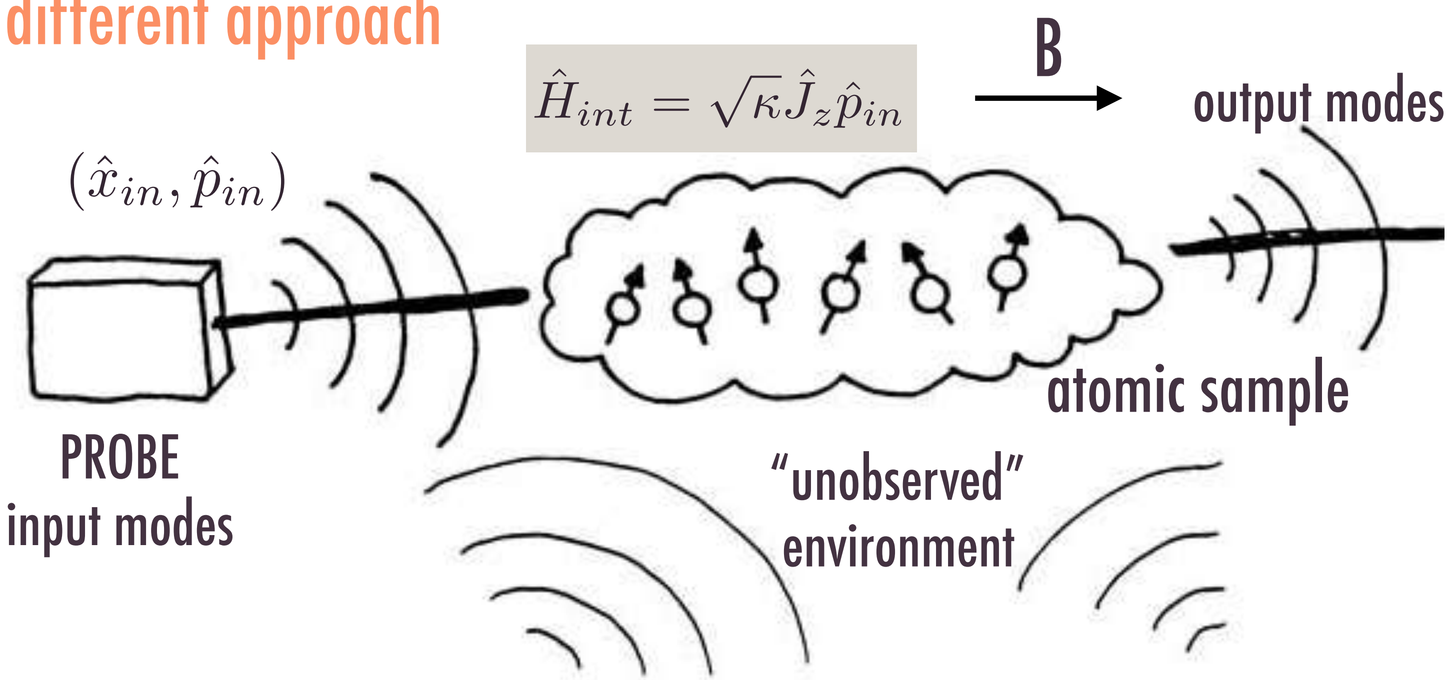
super-classical scaling can be recovered, by optimizing over the "interrogation time" of each experimental run

✓ GHZ initial state: $\mathcal{Q}_\omega \sim N^{5/3}$ with $t_{\text{opt}} \sim N^{-1/3}$

✓ Spin-Squeezed initial state: $\mathcal{Q}_\omega \sim N^{5/4}$ with $t_{\text{opt}} \sim N^{-1/8}$

Time-continuous measurements for quantum magnetometry

...a different approach



✓ atomic sample initially prepared in a spin-coherent state $|J, J\rangle$

✓ atomic sample coupled to a electromagnetic mode corresponding to either

a cavity mode in a strongly driven cavity

a far-detunted traveling mode passing through the ensemble

Thomsen et al., PRA (2002)

Madsen & Molmer, PRA (2004)

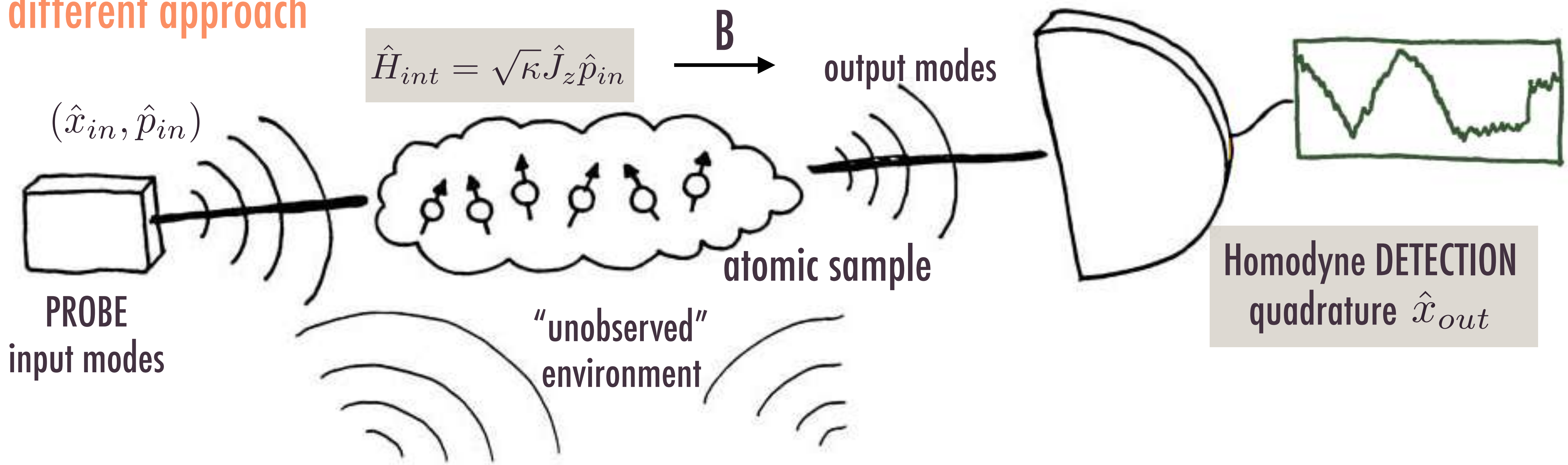
If the “output modes” are not measured, one obtains the dissipative master equation

$$\dot{\varrho} = -i\omega[\hat{J}_y, \varrho] + \kappa\mathcal{D}[\hat{J}_z]\varrho$$

transversal noise

Time-continuous measurements for quantum magnetometry

...a different approach



- ✓ atomic sample initially prepared in a spin-coherent state $|J, J\rangle_x$
- ✓ atomic sample coupled to a electromagnetic mode corresponding to either
  - a cavity mode in a strongly driven cavity
  - a far-detunted traveling mode passing through the ensemble
- ✓ time-continuous homodyne detection on the "output modes" with efficiency η

Thomsen et al., PRA (2002)

Madsen & Molmer, PRA (2004)

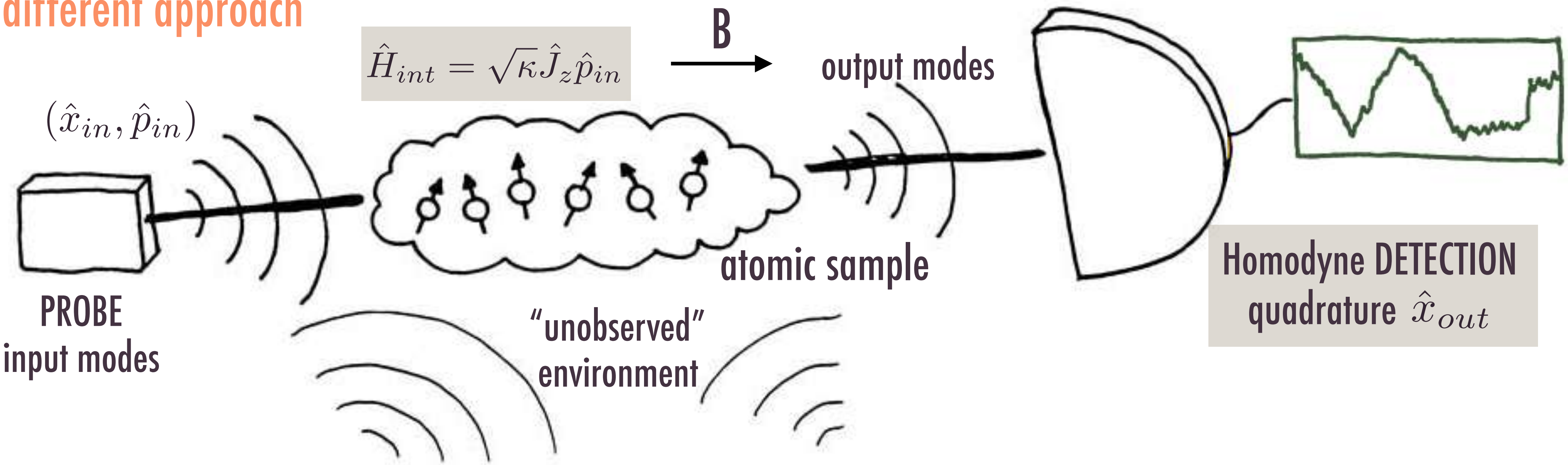
$$d\varrho_c = -i\omega[\hat{J}_y, \varrho_c]\, dt + \kappa\mathcal{D}[\hat{J}_z]\varrho_c\, dt + \sqrt{\eta\kappa}\mathcal{H}[\hat{J}_z]\varrho_c\, dw$$

$$dy_t = 2\sqrt{\eta\kappa}\,\mathrm{Tr}[\varrho_c\hat{J}_z]\, dt + dw$$

- ✓ continuous monitoring gives **information** on $\hat{J}_z$ and thus on the rotation due to magnetic field **B**
- ✓ conditional dynamics **squeeze** the variance $\Delta\hat{J}_z^2$

Time-continuous measurements for quantum magnetometry

...a different approach



- ✓ atomic sample initially prepared in a spin-coherent state $|J, J\rangle_x$
- ✓ atomic sample coupled to a electromagnetic mode corresponding to either
  - a cavity mode in a strongly driven cavity
  - a far-detunted traveling mode passing through the ensemble

Thomsen et al., PRA (2002)

Madsen & Molmer, PRA (2004)

- ✓ time-continuous homodyne detection on the "output modes" with efficiency η

$$d\varrho_c = -i\omega[\hat{J}_y, \varrho_c]\, dt + \kappa\mathcal{D}[\hat{J}_z]\varrho_c\, dt + \sqrt{\eta\kappa}\mathcal{H}[\hat{J}_z]\varrho_c\, dw$$

$$dy_t = 2\sqrt{\eta\kappa}\,\mathrm{Tr}[\varrho_c \hat{J}_z]\, dt + dw$$

- ✓ continuous monitoring gives **information** on $\hat{J}_z$ and thus on the rotation due to magnetic field **B**
- ✓ conditional dynamics **squeeze** the variance $\Delta\hat{J}_z^2$

Can we recover the Heisenberg limit ?

some results suggest that we can!

Geremia et al., PRL (2003)

Molmer & Madsen, PRA (2004)

Time-continuous measurements for quantum magnetometry

Information on the magentic field **B** can be obtained from two "sources" ...

- the classical photocurrent y_t corresponding to the monitoring the output field

  whose performance is quantified by the classical Fisher information

$$\mathcal{F}[p(y_t|\omega)] = \mathbb{E}_{dw}[(\partial_\theta \log \mathrm{Tr}[\tilde{\varrho}_c])^2]$$

Genoni, PRA **95** (2017)

- a final "strong" measurement on the conditional state ϱ_c

  whose performance is quantified by the quantum Fisher information

$$\mathcal{Q}_\omega[\varrho_c] = 2\sum_{jk} \frac{|\langle\psi_j|\partial_\omega \varrho_c|\psi_k\rangle|^2}{\lambda_j + \lambda_k}$$

Paris, IJQI **7** (2009)

Time-continuous measurements for quantum magnetometry

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Genoni, PRA **95** (2017)

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$$Q_\omega[\varrho_c] = 2 \sum_{jk} \frac{|\langle \psi_j | \partial_\omega \varrho_c | \psi_k \rangle|^2}{\lambda_j + \lambda_k}$$

QI **7** (2009)

How do we have to “combine” the two Fisher informations $\mathcal{F}[p(y_t|\omega)]$ and $Q_\omega[\varrho_c]$ to obtain the correct Quantum Cramér-Rao bound for our measurement strategy ?

Time-continuous measurements for quantum magnetometry

Cramér-Rao bound for strategies based on time-continuous measurements

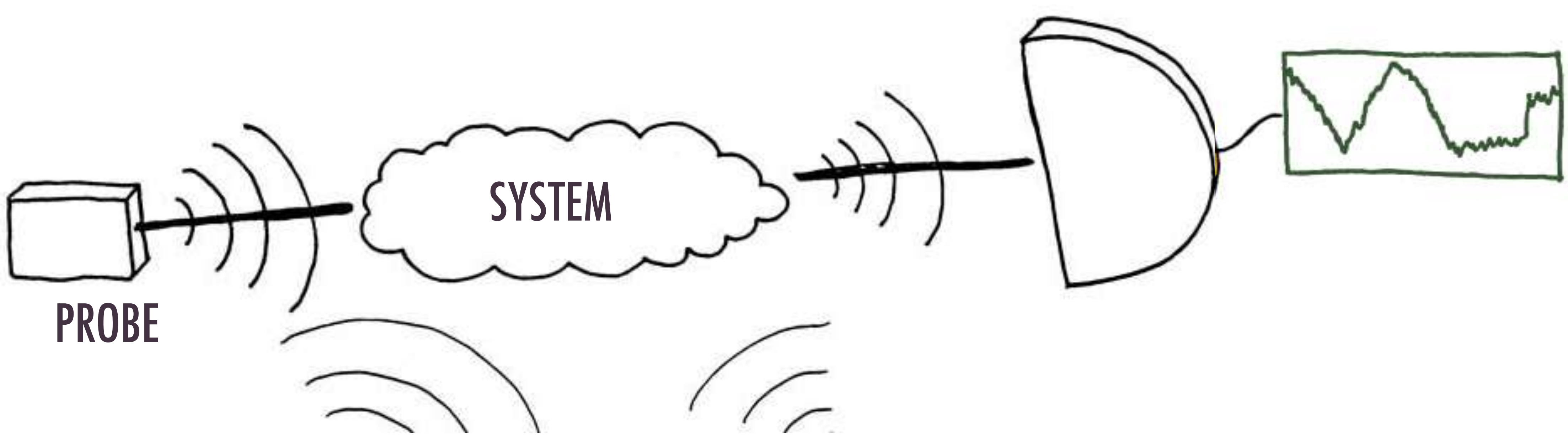
$$\mathrm{Var}_{\hat{\omega}}(\omega) \geq \frac{1}{M\,\{\mathcal{F}[p(y_t|\omega)] + \mathbb{E}_{dw}[\mathcal{Q}_\omega[\varrho_c]]\}}$$

Albarelli et al., arXiv:1706:00485

the quantity that correctly quantifies the performances of our measurement strategy, is the effective QFI

$$\tilde{Q}_\omega = \mathcal{F}[p(y_t|\omega)] + \mathbb{E}_{dw}[\mathcal{Q}_\omega[\varrho_c]]$$

classical Fisher Information of the photocurrent + average over trajectories Quantum Fisher Information of the conditional state



Effective Quantum Fisher Information

...in the limit of large spin **J** we can analytically evaluate the effective QFI

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Genoni, PRA **95**, 012116 (2017)

<u>Classical FI corresponding to the photocurrent</u>

$$\mathcal{F}[p(y_t|B)] = 2\eta\kappa J e^{-\kappa t/2} \left(\mathbb{E}_{dw} \left[\left(\partial_B \langle \hat{P}(t) \rangle_c \right)^2 \right] \right)$$

analytical and deterministic solution

Effective Quantum Fisher Information

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$$\mathcal{F}[p(y_t|B)] = 2\eta\kappa J e^{-\kappa t/2}\left(\cancel{\mathbb{E}_{dw}}\left[\left(\partial_B\langle\hat{P}(t)\rangle_c\right)^2\right]\right)$$

$$= \frac{64\gamma^2\eta J^2 e^{\kappa(-t)}\left(e^{\frac{\kappa t}{4}}-1\right)^3}{9\kappa^2\left((4\eta J+1)e^{\frac{\kappa t}{2}}-4\eta J\right)}\cdot$$

$$\cdot\left(-4\eta J - 12\eta J e^{\frac{\kappa t}{4}} + 3(4\eta J+3)e^{\frac{\kappa t}{2}} + (4\eta J+3)e^{\frac{3\kappa t}{4}}\right).$$

Effective Quantum Fisher Information

...in the limit of large spin **J** we can analytically evaluate the effective QFI

$$\tilde{Q}_\omega = \mathcal{F}[p(y_t|\omega)] + \mathbb{E}_{dw}[\mathcal{Q}_\omega[\varrho_c]]$$

<u>Average QFI of the conditional state</u>

Pinel et al., PRA **88**, 040102 (2013)

$$\mathbb{E}_{dw}\left[\mathcal{Q}[\varrho^{(c)}]\right] = \mathbb{E}_{dw}\left[\frac{\left(\partial_B \langle \hat{P}(t)\rangle_c\right)^2}{\mathrm{Var}_c[\hat{P}(t)]}\right]$$

analytical and deterministic solutions
for both quantities

Effective Quantum Fisher Information

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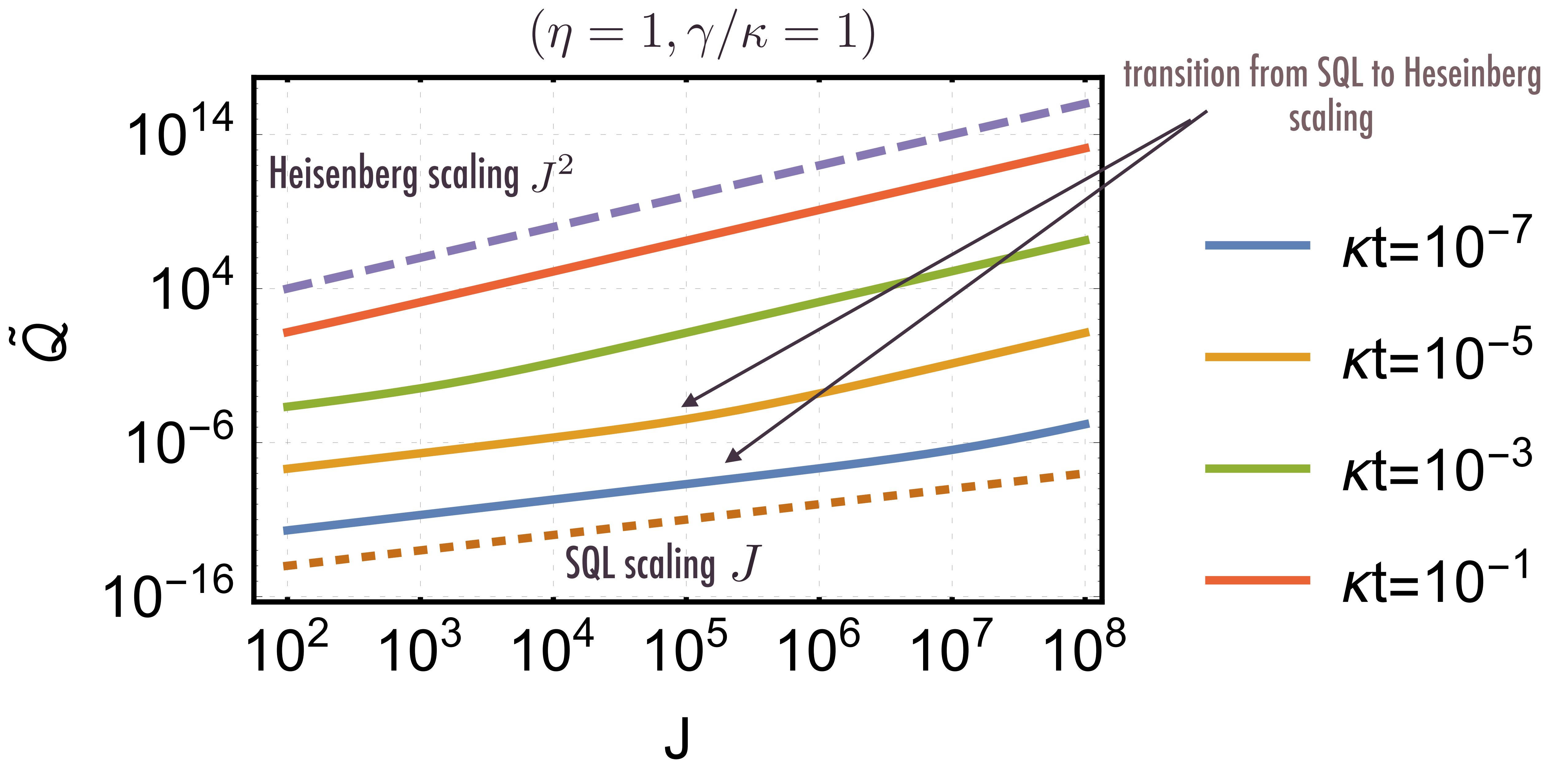
$$= \frac{32\gamma^2 J\left(12\eta J - 4\eta J e^{-\frac{\kappa t}{2}} + \frac{-8\eta J - 3}{\sqrt{e^{-\frac{\kappa t}{2}}}} + 3\right)^2}{9\kappa^2\left[(4\eta J + 1)e^{\frac{\kappa t}{2}} - 4\eta J\right]}$$

Results

J - SCALING (with perfect detectors)

Albarelli et al., arXiv:1706:00485



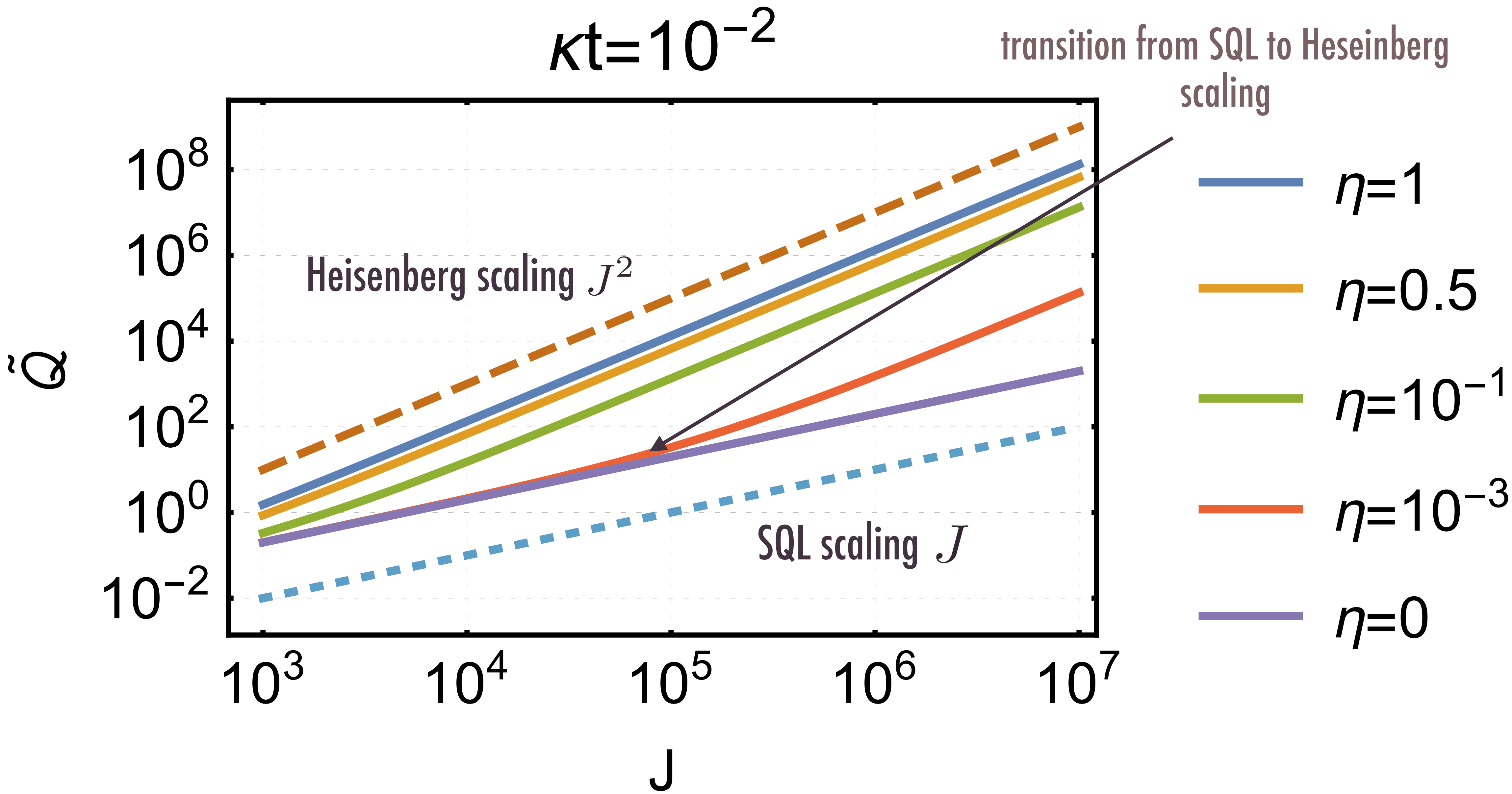
HEISENBERG LIMIT RECOVERED !

...for spin J and measurement time t large enough...

Results

Role of measurement efficiency

Albarelli et al., arXiv:1706:00485



HEISENBERG LIMIT RECOVERED also for non-unity efficiency !

...measurement with **efficiency smaller** than **one**, simply imply to need **larger values** of **J** to observe the **transition** from **SQL** to **HL** scaling...

Results

Albarelli et al., arXiv:1706:00485

Can we do better than this ?

"ultimate" QFI

$$\tilde{\mathcal{Q}}_\omega = \mathcal{F}[p(y_t|\omega)] + \mathbb{E}_{dw}[\mathcal{Q}_\omega[\varrho_c]] \leq \overline{\mathcal{Q}}_\omega$$

takes into account all the possible measurement that quantum mechanics allows to perform on system+environment

Gammelmark et al., PRL (2014)

Results

Albarelli et al., arXiv:1706:00485

Can we do better than this ?

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takes into account all the possible measurement that quantum mechanics allows to perform on system+environment

Gammelmark et al., PRL (2014)

For our strategy with **perfectly efficient detectors**

$$\tilde{\mathcal{Q}}_\omega(\eta = 1) = \overline{\mathcal{Q}}_\omega$$

!!! OPTIMAL STRATEGY !!!

in the presence of transversal noise
there's no need of performing more complicated strategies !

Conclusions & Outlooks

- Time-continuous measurements as a tool to generate probes for quantum metrology
  - ✓ quantum state engineering of squeezed states for levitating nanospheres trapped in optical cavities

Genoni, Zhang, Millen, Barker & Serafini, NJP **17** (2015)

- Time-continuous measurement as a new tool for noisy quantum metrology
  - ✓ Heisenberg-scaling recovered for noisy magnetometry
  - ✓ unit measurement efficiency not necessary to observe the quantum enhancement
  - ✓ optimality of our strategy for magnetometry with transversal noise

Genoni, PRA **95**, 012116 (2017)

Albarelli, Rossi, Paris & Genoni, arXiv:1706.00485

- Outlooks
  - ✓ what about other kind of noise ?
  - ✓ what about other quantum metrology applications?
  - ✓ what about multi-parameter metrology ?

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Quantum Control for Advanced Quantum Metrology

MARIE CURIE FELLOWSHIP ConAQuMe
(Grant agreement 701154)

Quantum probes for complex systems

EU STREP PROACTIVE QuProCS
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