QTLab presents:

All-optical quantum simulator of qubit noisy channels

S. Cialdi[1], M. A. C. Rossi[1], C. Benedetti[1], B. Vacchini[1,2], D. Tamascelli[1], S. Olivares[1] and M. G. A. Paris[1]

[1]Quantum Technology Lab, Dipartimento di Fisica, Università degli Studi di Milano, I-20133, Milano, Italy
[2]INFN, Sezione di Milano, I-20133 Milano, Italy

Website: http://users.unimi.it/aqm

We suggest and demonstrate an all-optical quantum simulator for single-qubit noisy channels originating from the interaction with a fluctuating field. The simulator employs the polarization degree of freedom of a single photon and exploits its spectral components to average over the realizations of the stochastic dynamics. As a proof of principle, we run simulations of dephasing channels driven either by Gaussian (Ornstein-Uhlenbeck) or non-Gaussian (random telegraph) stochastic processes. [http://dx.doi.org/10.1063/1.4977023]

Appl. Phys. Lett. 110, 081107 (2017)

The Physical Model

We simulate the evolution of a single qubit evolving under a time-dependent Hamiltonian of the form

$$H(t) = H_0 + H_{\text{int}}(t) = \varepsilon\sigma_z + X(t)\sigma_z$$

σ_z is the Pauli matrix, ε the energy splitting of the qubit and $X(t)$ an arbitrary real-valued continuous-time **stochastic** process.

Initial state: $|\psi_0\rangle = |+\rangle \equiv \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $\rho_0 = |\psi_0\rangle\langle\psi_0|$

Evolution operator and phase for each **realization**:

$$U(t) = e^{-i\int_0^t H(\tau)d\tau} \qquad \Phi_r(t) = \int_0^t X_r(\tau)d\tau$$

Evolved state: $|\psi(t)\rangle = \frac{1}{\sqrt{2}}(e^{-2i\Phi_r(t)}|0\rangle + |1\rangle)$.

Ensemble average: $\rho(t) = \langle U(t)\rho_0 U(t)^\dagger\rangle_{\{X(t)\}}$

In the interaction picture: $\rho_S(t) = \frac{1}{2}\begin{pmatrix} 1 & \langle e^{-2i\Phi_r(t)}\rangle \\ \langle e^{2i\Phi_r(t)}\rangle & 1 \end{pmatrix}$

For the **Random Telegraph Noise** (**RTN**), the realization $X_r(t)$ flips randomly between the values ± 1 with a switching rate γ. The initial values $X_r(0)$ are selected randomly with equal probability between ± 1 for each pixel.

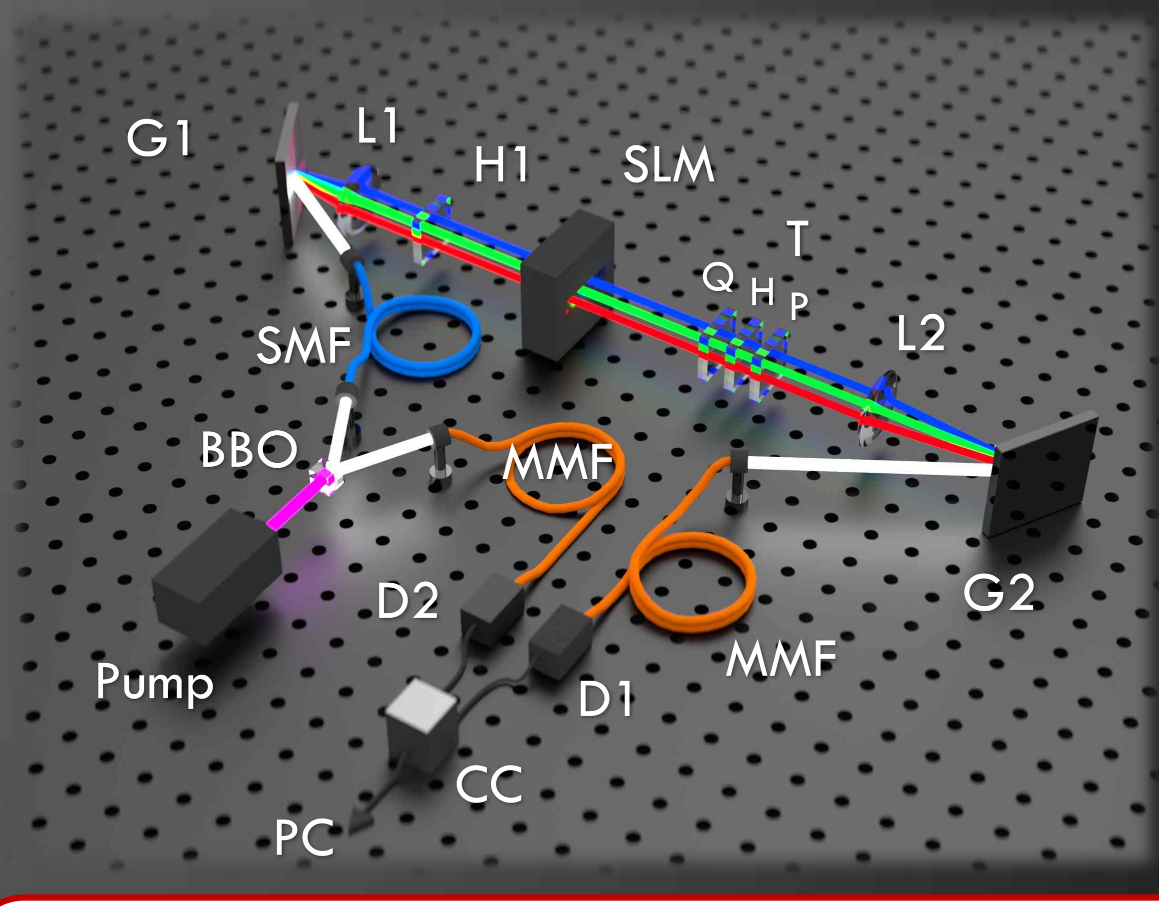
For the **Ornstein-Uhlenbeck** (**OU**) process, instead, we have

$$X_r(\bar{t} + \delta\bar{t}) = (1 + 2\gamma\,\delta\bar{t})X_r(\bar{t}) + 2\sqrt{\gamma}\,dW(\bar{t})$$

where $dW(\bar{t})$ is a Wiener increment with the mean equal zero and standard deviation $\sigma = \sqrt{\delta\bar{t}}$ and $\bar{t}$ the simulation time. For each realization we impose the initial condition $X_r(0) = 0$.

The Experimental Implementation

We realized an experimental all-optical setup that allows us to obtain the evolved state upon the generation of n sample-paths in a single run. The quantum information carrier is a photon. The **polarization** of the photon is used to encode the state of a qubit, whereas its **spectral components** are exploited to implement the trajectories of the stochastic process describing the fluctuating field.



diode **pump** laser @405.5nm using a **BBO** crystal (1mm thick);
SMF: single-spatial-mode and polarization preserving fiber;
MMF: multimode fiber;
G1-G2: gratings (1714 lines/mm);
L1-L2: lens(f=500mm);
H1, half-wave-plate;
SLM: spatial light modulator (640 pixels, 100 μm/pixel);
T, tomographic apparatus;
Q: quarter-wave plate;
P, polarizer;
D1-D2: single photon detectors;
CC: coincidences counter. The acquisition time is of 10s for each measure of coincidence counts.
The inset shows the measured PDC spectrum.

The SLM controlled by the computer is used to imprint a different phase $\Phi_r(t)$ for each pixel $|\eta_r\rangle$ on the horizontal polarization component:

$$U(\bar{t}) = \exp\left[-2\mathrm{i}|H\rangle\langle H| \otimes \sum_r \Phi_{r(t)}|\eta_r\rangle\langle\eta_r|\right].$$

Taking the marginal, one obtain the wanted $\rho_{S,\text{exp}}(t) = \frac{1}{2n}\sum_{r=1}^{n}\begin{pmatrix} 1 & e^{-2i\Phi_r(t)} \\ e^{2i\Phi_r(t)} & 1 \end{pmatrix}$.

Due to the **imperfections** of the experimental apparatus, in each realization, the state may not exactly pure but rather of the form

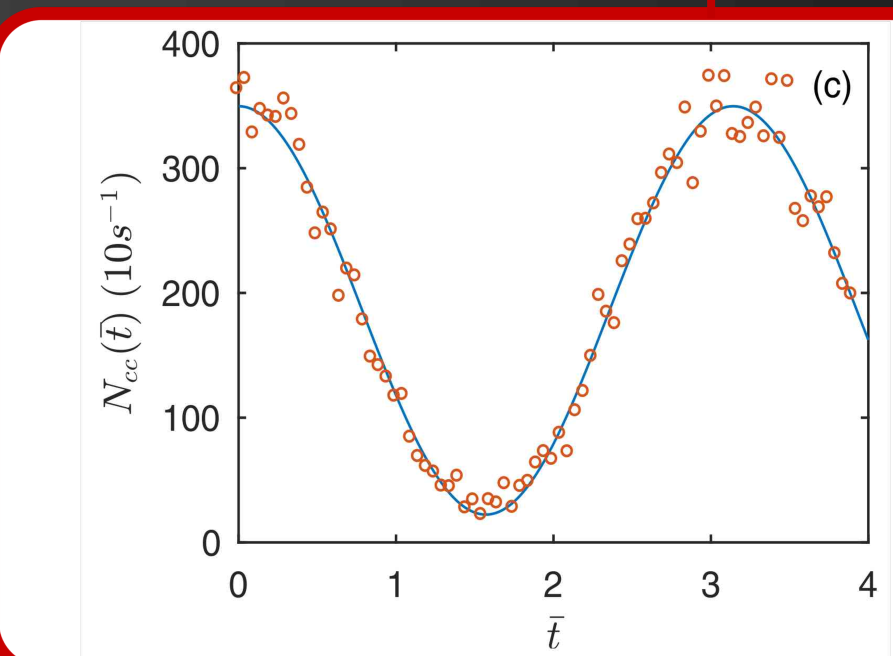
$$\rho_S = p\,\rho_{S,\text{exp}} + (1-p)\rho_{\text{mix}}$$

where $\rho_{\text{mix}} = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|)$. The relevant quantity we want to measure is

$$C(t) = \langle H|\rho_{S,\text{exp}}(t)|V\rangle = \frac{1}{2}\langle e^{-2i\Phi_r(t)}\rangle$$

The Simulations

The average over the realizations of the noise is performed in parallel by (coherently) collecting the different spatial components $|\omega\rangle$ through the lens L2 and the grating G2 into a multimode fiber.

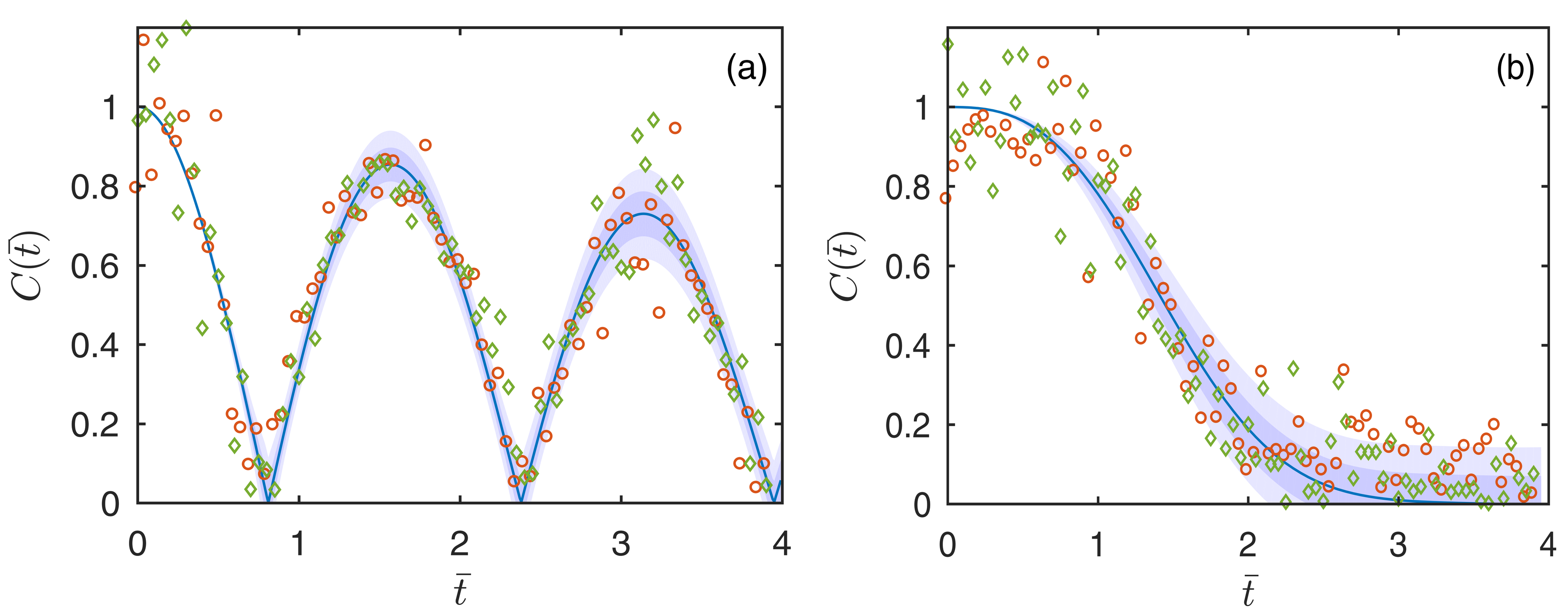


In order to obtain p, we measure the RTN at $\gamma = 0$, since in this case $\left\langle e^{-2i\Phi_r(\bar{t})}\right\rangle = \cos(2\bar{t})$.
We find $p = 0.88 \pm 0.002$

Coincidence counts in the case of RTN with $\gamma = 0$, and the blue line is the fit with the function

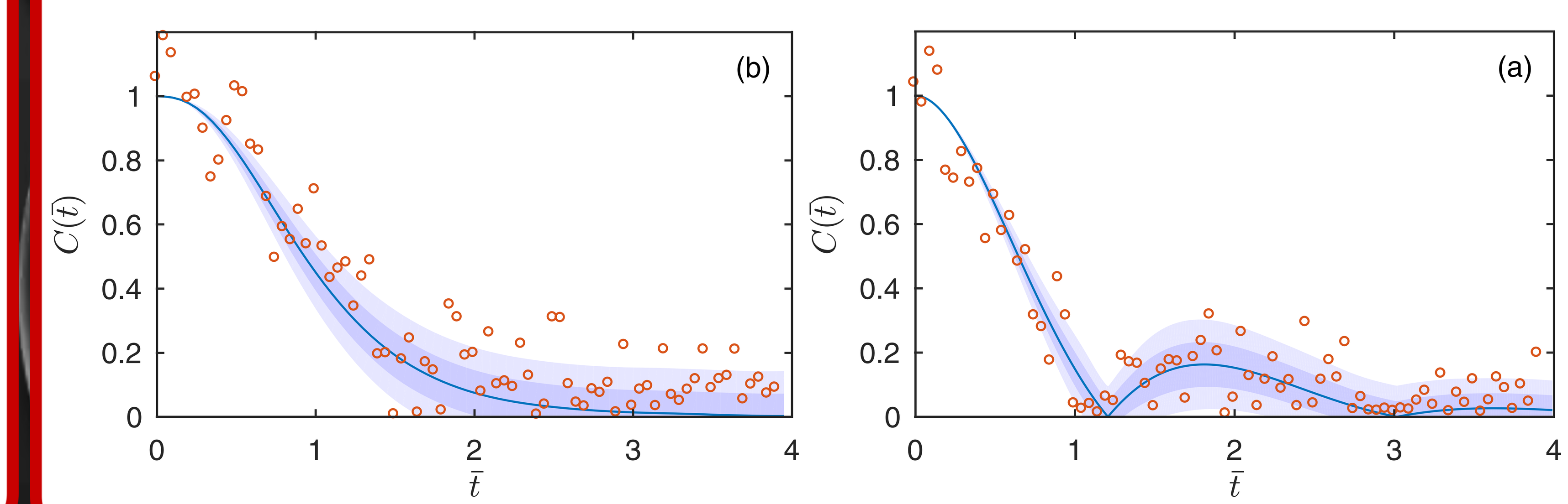
$$N_{cc} = N(1 + p\cos(2t))$$

○ **Tomography** by performing four projective measurements
vs
◇ **Projection** onto the $|+\rangle$ state $\langle +|\rho_{S,\text{exp}}|+\rangle = \frac{1}{2}\left(1 + p\,\text{Re}\left\langle e^{-2i\Phi_r(\bar{t})}\right\rangle_n\right)$



Dynamics of $C(t)$ for RTN (a) and OU (b) with $\gamma = 0.1$. Red circles and green diamonds represent the data obtained, respectively, with tomographic reconstruction of $\rho_{S,\text{exp}}$ and projection onto the state $|+\rangle$. The blue line is the analytic solution. The shades represent intervals of 1σ (darker) and 2σ (lighter) around the analytic solution, where σ is the standard deviation of paths obtained with 100 realizations of the stochastic process. Note that the noise for small $\bar{t}$ is due to the Poissonian fluctuations on the coincidence counts.

Dynamics of $C(t)$ evaluated by the method of the projection onto the state $|+\rangle$ in the case with $\gamma = 1$ for RTN (a) and OU (b) stochastic process. The blue line is the analytic solution and the blue shades represent intervals of 1 σ (darker) and 2 σ (lighter) around the analytical solution, where σ is the standard deviation of paths obtained with 100 realizations of the stochastic process.



Refs:
C. Benedetti, F. Buscemi, P. Bordone, M. G. A. Paris Int. J. Quantum Inform. **10**, 1241005 (2012).
C. Benedetti, M. G. A. Paris, and S. Maniscalco, Phys. Rev. A **89**, 012114 (2014).
S. Cialdi, M. A. C. Rossi, C. Benedetti, B. Vacchini, D. Tamascelli, S. Olivares and M. G. A. Paris, *Appl. Phys. Lett.* **110**, 081107 (2017).
M. A. C. Rossi, C. Benedetti, S. Cialdi, D. Tamascelli, S. Olivares, B. Vacchini, and M. G. A. Paris, arXiv: 1705.05852 (2017).

Sponsored by:
QuProCS
UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI FISICA