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DIPARTIMENTO DI FISICA

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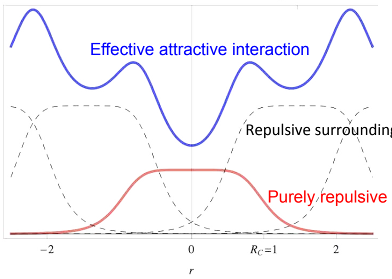
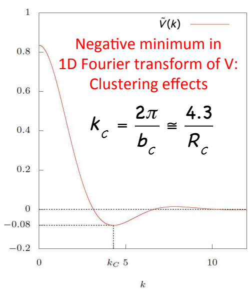
1D soft Bosons across the liquid/cluster-liquid transition: the interplay between Luttinger and quantum Ising universality classes

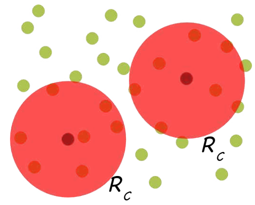
Model: 1D system of Bosons interacting via a soft shoulder potential, typical of dressed Rydberg gases. Previous investigations:

- classical [Phys. Rev. E 92, 022138 (2015)]
- quantum lattice model [Phys. Rev. Lett. 111, 165302 (2013)].

$$\hat{H} = \hat{T} + \hat{V} = \sum_{i=1}^{N} \frac{\hat{p}_i^2}{2m} + \sum_{i<j=1}^{N} \frac{V_0}{\left|\hat{x}_i - \hat{x}_j\right|^6 + R_c^6}$$

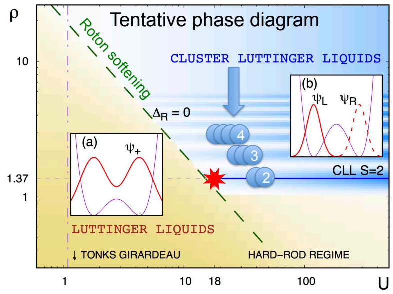






Rydberg state: highly excited electronic state
→ R_C Rydberg blockade radius
Resonant light-atom coupling:
full blockade→ hard-core effective particles

Dressed Rydberg state:
Off-resonant light-atom coupling:
quantum superposition
of ground-state and Rydberg state
→ soft-core effective particles



Natural units:

- Lenght: R_c
- Energy: $E_C = \hbar^2/mR_c^2$

$\Rightarrow U = V_0/E_c$
$\Rightarrow \rho = N R_c/L$

Tentative phase diagram (log-log scale): A star marks the critical point between the Luttinger-Liquid (LL) and Cluster-Luttinger-Liquid (CLL) phases for densities commensurate to 2-particle clusters. The dashed line corresponds to the softening of the roton in the Bogoliubov approximation of the LL phase. Inset (a) represents the delocalized orbital in the effective double-well potential experienced in the homogeneous phase. Inset (b) represents the localized left and right orbitals.

Luttinger Liquid (LL): Universality class of 1D systems, characterized by a gapless bosonic phonon mode at small momenta. The low-energy and momentum Physics is governed by the hydrodynamic Hamiltonian: ⟶

$$\hat{H}_{LL} = \frac{\hbar v_s}{2\pi} \int dx \left\{ K_L \left[\nabla\hat{\theta}(x)\right]^2 + \frac{1}{K_L}\left[\nabla\phi(x)\right]^2 \right\}$$

A large Luttinger parameter $K_L > 1$ favors the fluctuation of the counting field $\phi(x)$, i.e. liquid-like behavior, while small values of K_L induce crystal-like behavior, by disordering the phase field $\theta(x)$.

v_S = sound velocity
K_L = LL parameter

$$\hat{\Psi}^{+}(x) \approx \sqrt{\rho_0 + \tfrac{1}{\pi}\nabla\phi(x)}\, e^{-i\theta(x)}$$

Methods: We employ quantum Monte Carlo (MC) simulations, which allow for the exact calculation of imaginary-time correlations $f_q(\tau)$, and a stochastic analytic continuation method [Phys. Rev. B 82, 174510 (2010)], to extract the dynamical structure factor $S(q,\omega)$.

$$f_q(\tau) = \left\langle \rho_q(\tau)\rho_{-q}(0)\right\rangle = \frac{\left\langle \psi_0 \left| e^{\tau\hat{H}} \left(\frac{1}{\sqrt{N}}\sum_{j=1}^{N} e^{-iqx_j}\right) e^{-\tau\hat{H}} \left(\frac{1}{\sqrt{N}}\sum_{j=1}^{N} e^{iqx_j}\right) \right| \psi_0 \right\rangle}{\left\langle \psi_0 | \psi_0 \right\rangle} = \int_0^\infty d\omega\, e^{-\omega\tau} S(q,\omega)$$

Imaginary-time density-density correlation — Dynamic structure factor

Note: extracting $S(q,\omega)$ from MC calculation of $f_q(\tau)$ is an ill-posed inverse problem
⇒ regularization and statistical analysis needed!

The Path Integral Ground State (PIGS) MC method [J. Chem. Phys. 113, (2000)] is an "exact" projector technique that provides direct access to ground-state expectation values of Bosonic systems, given the microscopic Hamiltonian $\hat{H}$. Observables are calculated as:

$$\left\langle \hat{O} \right\rangle = \frac{\left\langle \psi_0 \left| \hat{O} \right| \psi_0 \right\rangle}{\left\langle \psi_0 | \psi_0 \right\rangle} \cong \lim_{\tau>>1} \frac{\left\langle \psi_\tau \left| \hat{O} \right| \psi_\tau \right\rangle}{\left\langle \psi_\tau | \psi_\tau \right\rangle}$$

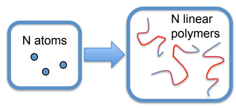
$$\left|\psi_\tau\right\rangle = e^{-\tau\hat{H}}\left|\psi_T\right\rangle \underset{\tau>>1}{\overset{\cong}{\lim}} \left|\psi_0\right\rangle$$

The imaginary-time projection of an initial "trial" state $|\psi_T\rangle$ (provided non-orthogonality to the ground state) converge on the true ground state $|\psi_0\rangle$

The Path Integral Ground State method uses a Trotter's decomposition to manage the imaginary time evolution via path integration:

$$e^{-\tau\hat{H}} = \left(e^{-\frac{\tau}{M}\hat{H}}\right)^M \underset{M>>1}{\cong} \left(e^{-\frac{\tau}{2M}\hat{V}} e^{-\frac{\tau}{M}\hat{T}} e^{-\frac{\tau}{2M}\hat{V}}\right)^M$$

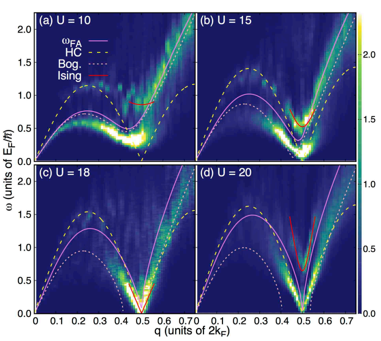
⇒ quantum-classical mapping:
the quantum system is mapped onto a classical system of special interacting linear polymers



Results: At low density, the system maps to the Lieb-Liniger (1D contact interaction), Tonks-Giradeau (TG: 1D ideal Fermi gas), and 1D Hard-rods (HR) models. In the weakly-interacting homogeneous regime, a rotonic spectrum marks the tendency to clusterization. With strong interactions, we indeed observe cluster liquid phases emerging, characterized by the spectrum of a composite harmonic chain. Luttinger theory has to be adapted by changing the reference lattice density field. We find convincing evidence of a secondary mode, which becomes gapless only at the transition, and associate it to a transverse Ising model [Science 327, 177 (2010)], related to the instability of the reference lattice density field towards coalescence of sites.

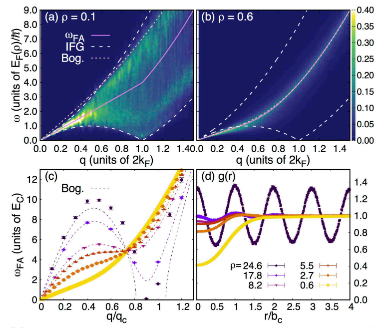
Transverse Ising (TI) model:

$$\hat{H} = -J\sum_{i=1}^{N} \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z - h\sum_{i=1}^{N} \hat{\sigma}_i^x$$



$$\varepsilon_{TI}(q) = \sqrt{(J-h)^2 + 4Jh\left(\sin\frac{qb_c}{2}\right)^2}$$

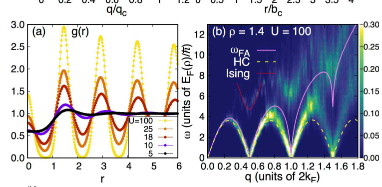
Spectra at ρ=1.37 along the transition for different values of U, compared to the Feynman's approximation, the Bogoliubov spectrum.
At $q=k_F=q_C$ the secondary mode is fitted with an Ising-like spectrum.



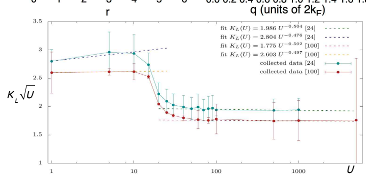
Spectra in the liquid phase at U = 1.09, close to the TG regime (a) and at intermediate ρ (b) (values beyond scale are plotted in white).
(c) Evolution of Feynman approximation for the dominant mode, from the liquid regime to the large-ρ cluster phase, compared to Bogoliubov approximation.
(d) Evolution of the pair distribution function.

Feynman spectrum: $\varepsilon_{FA}(q) = \hbar\omega_{FA} = \frac{\hbar^2 q^2}{2mS(q)}$

Bogoliubov spectrum: $\varepsilon_B(q) = \sqrt{\left(\frac{\hbar^2 q^2}{2m}\right)^2 + \frac{\hbar^2 q^2}{m}\rho\tilde{V}(q)}$



(a) Pair distribution function at ρ=1.37 from the LL to the CLL phases.
(b) Typical spectrum in the CLL phase.



Luttinger parameter (scaled by $\sqrt{U}$) across the transition

$$K_L = \lim_{q\to 0} \frac{2\pi\rho S(q)}{q}$$