Optical quantum communication with PNR detectors: phase estimation and coherent-state discrimination in the presence of phase noise

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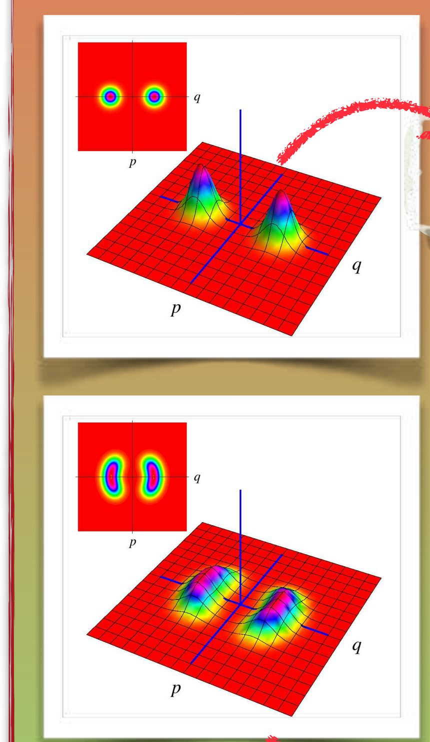
State discrimination protocols in communication channels with phase-shifted coherent states are based on interferometric schemes. In these setups the signals are mixed with a local oscillator (LO) with a well-defined reference phase. However the coherent signals from the sender to the receiver can be strongly affected by both phase drift and phase noise, making the communication impracticable. Therefore it is crucial to monitor the relative optical phase between signal and LO. Recent technological progresses brought to commercialization of photon-number-resolving (PNR) detectors, capable of distinguishing among different Fock states, up to a detecting threshold. These detectors provide additional information, giving direct access to the photon statistics of the output states. We show how the use of PNR detectors may improve the estimation of phase drifts in a Kennedy-like receiver, employing a Bayesian post-processing of a small amount of data drawn from the outputs of the shot-by-shot discrimination protocol. We also propose a homodyne-like detection scheme involving PNR detectors to discriminate between the coherent signals affected by either uniform or gaussian phase noise, showing how this strategy well approaches the Helstrom bound. Both the estimation and discrimination protocols are validated by proof-of-principle experiments employing hybrid photodetectors.

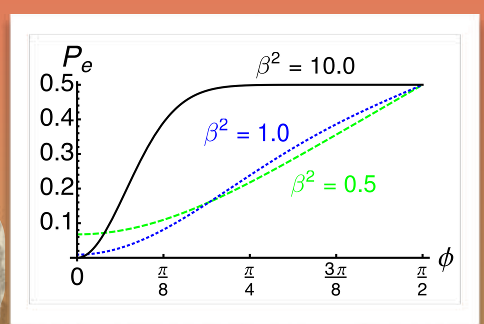
ESTIMATION OF PHASE DRIFT IN THE PRESENCE OF PHASE NOISE

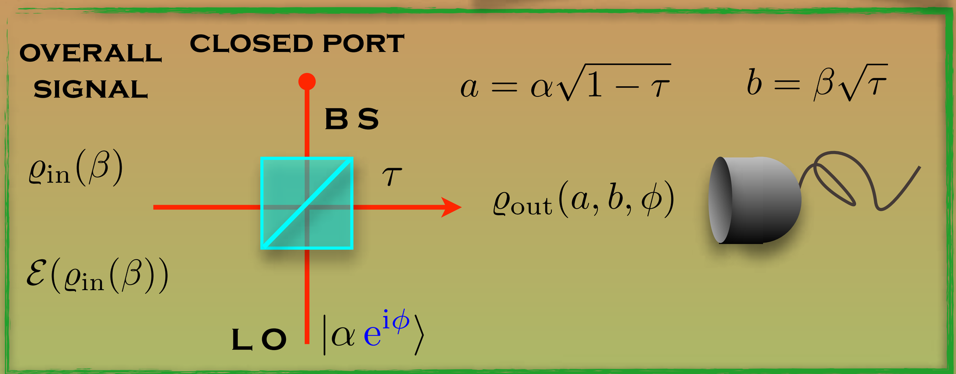
THE SCHEME

(KENNEDY - LIKE RECEIVER)



$$\varrho_{\rm in}(\beta) = \frac{|\beta\rangle\langle\beta| + |-\beta\rangle\langle-\beta|}{2}$$





$a = \alpha\sqrt{1-\tau}$ $\quad b = \beta\sqrt{\tau}$

$$\varrho_{\rm in}(\beta) \mapsto \mathcal{E}(\varrho_{\rm in}(\beta)) = \int_{\mathbb{R}} d\varphi\, f(\varphi)\hat{U}(\varphi)\varrho_{\rm in}(\beta)\hat{U}^\dagger(\varphi)$$

UNIFORM NOISE

$$f(\varphi) = \begin{cases} \frac{1}{\gamma} & \varphi \in [-\frac{\gamma}{2}, \frac{\gamma}{2}] \\ 0 & \varphi \notin [-\frac{\gamma}{2}, \frac{\gamma}{2}] \end{cases}$$

GAUSSIAN NOISE

$$f(\varphi) = \mathcal{N}(\varphi; 0, \sigma^2)$$

PHASE SHIFT

$$\hat{U}(\varphi) = \exp(-i\varphi\hat{a}^\dagger\hat{a})$$

DETECTION

ON/OFF DETECTOR

$$\Pi_0 = |0\rangle\langle 0|$$

$$\Pi_1 = \sum_{n>0} |n\rangle\langle n| = \mathbb{I} - \Pi_0$$

$$P_{\rm off} = e^{-(a^2+b^2)}\cosh(2ab\cos(\phi))$$

$$P_{\rm on} = 1 - P_{\rm off}$$

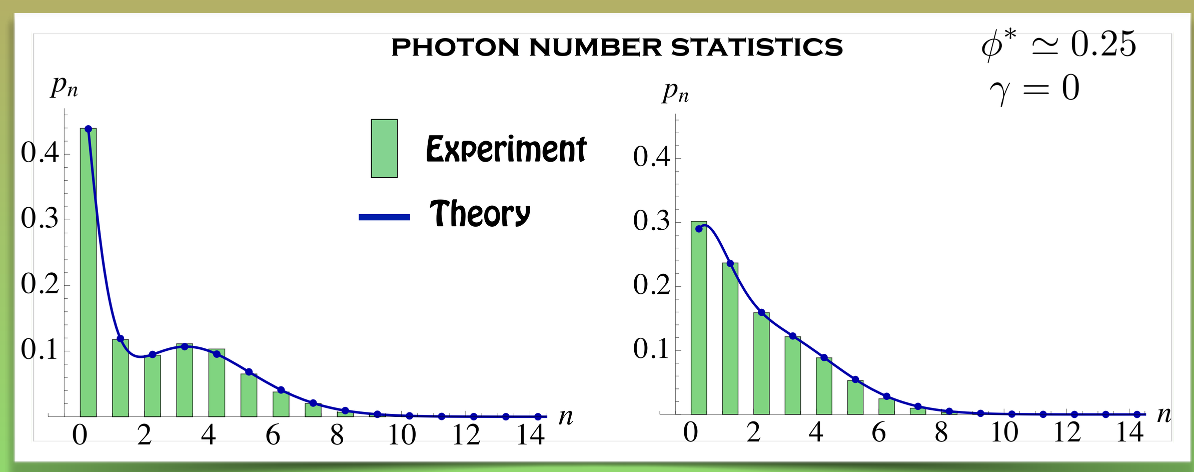
information on the relative phase ϕ

PNR DETECTOR

$$\Pi_n = |n\rangle\langle n|$$

$$\nu_\pm = a^2 + b^2 \pm 2\,a\,b\cos\phi$$

$$p_n(a,b,\phi) = \frac{1}{2}\left(e^{-\nu_+}\frac{\nu_+^n}{n!} + e^{-\nu_-}\frac{\nu_-^n}{n!}\right)$$



POST - PROCESSING

$$\{n_k\} = \{n_1, n_2, \ldots, n_M\}$$

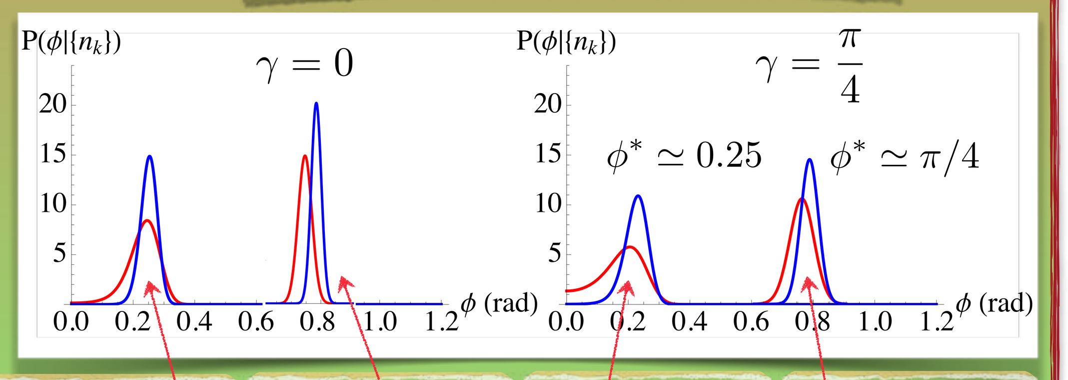
SAMPLE PROBABILITY $\quad P(\{n_k\}|\phi) = \prod_{n=0}^{\infty} p_n(a,b,\phi)^{m_n}$

BAYES THEOREM

POSTERIOR PROBABILITY $\quad P_{\rm PNR}(\phi|\{n_k\}) = \mathcal{N}_{\rm PNR}\, P(\{n_k\}|\phi)$

$$\bar{\phi} = \int d\phi\, \phi\, P_{\rm PNR}(\phi|\{n_k\})$$

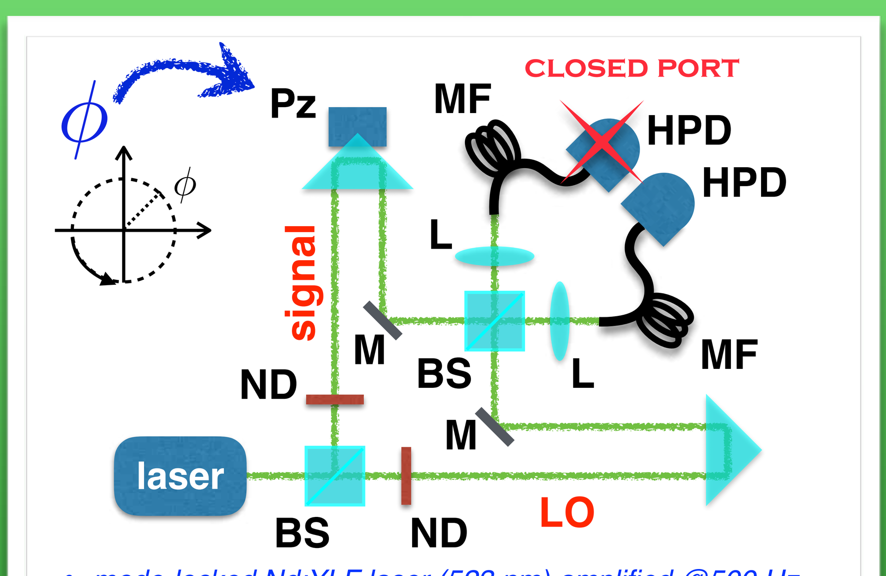
$$\mathrm{Var}_\phi = \int d\phi\, (\phi - \bar{\phi})^2\, P_{\rm PNR}(\phi|\{n_k\})$$



$\bar{\phi}_{\rm on/off} = 0.230 \pm 0.054$ | $\bar{\phi}_{\rm on/off} = 0.746 \pm 0.023$ | $\bar{\phi}_{\rm on/off} = 0.174 \pm 0.072$ | $\bar{\phi}_{\rm on/off} = 0.766 \pm 0.037$

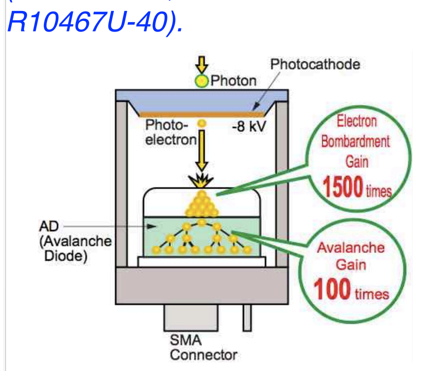
$\bar{\phi}_{\rm PNR} = 0.251 \pm 0.027$ | $\bar{\phi}_{\rm PNR} = 0.785 \pm 0.017$ | $\bar{\phi}_{\rm PNR} = 0.223 \pm 0.040$ | $\bar{\phi}_{\rm PNR} = 0.789 \pm 0.027$

EXPERIMENTAL SETUP



- mode-locked Nd:YLF laser (523 nm) amplified @500 Hz
- second-harmonic pulses (5 ps)
- variable neutral density filters (ND)
- multi-mode fiber with core diameter 600 μm

The scheme involves linear ***hybrid photodetectors*** *(Hamamatsu, mod. R10467U-40).*



- room temperature
- good quantum efficiency ($\approx$ 50 % @ 500 nm)
- negligible dark-count rate
- limited photon-counting capability
- linear response up to 100 photons

RESULTS

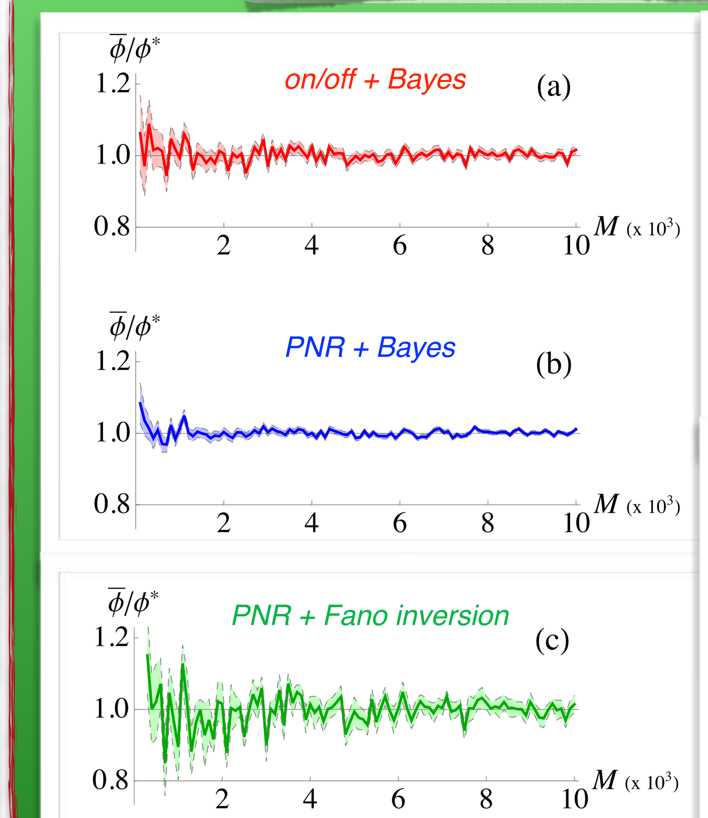
FANO FACTOR

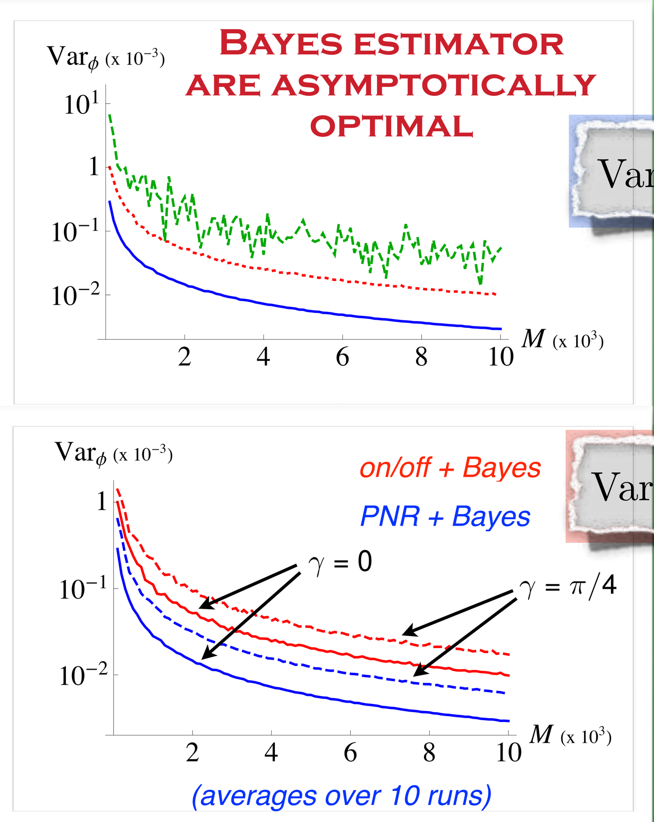
$$\mathcal{F}(\phi) = \mathrm{Var}[\hat{N}]/\langle\hat{N}\rangle = \frac{b^2 + 2a^2\left[\frac{1}{2} + b^2(1+\cos(2\phi))\right]}{b^2 + a^2}$$

$$\phi = \mathcal{F}^{-1}(\phi)$$

CRAMÉR - RAO BOUND

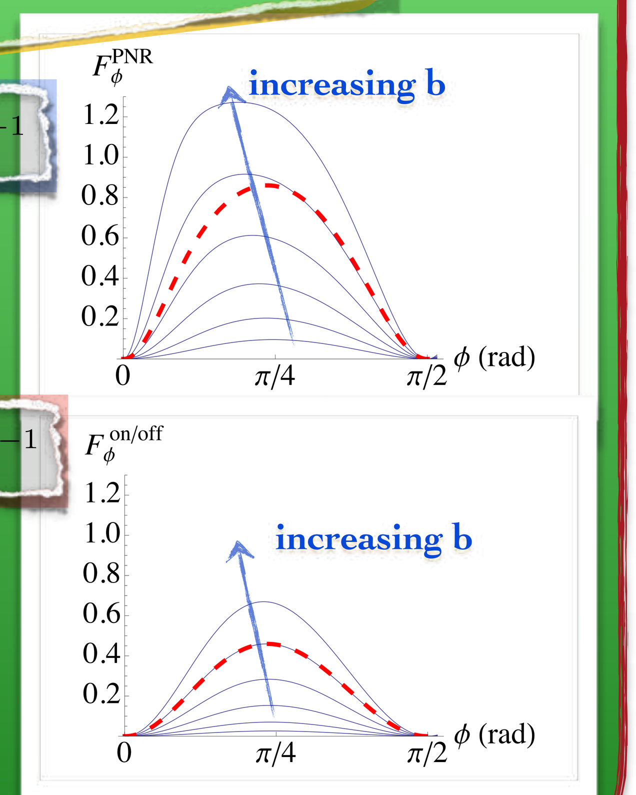
$$\mathrm{Var}(\phi) \geq \frac{1}{M\,F(\phi)}$$



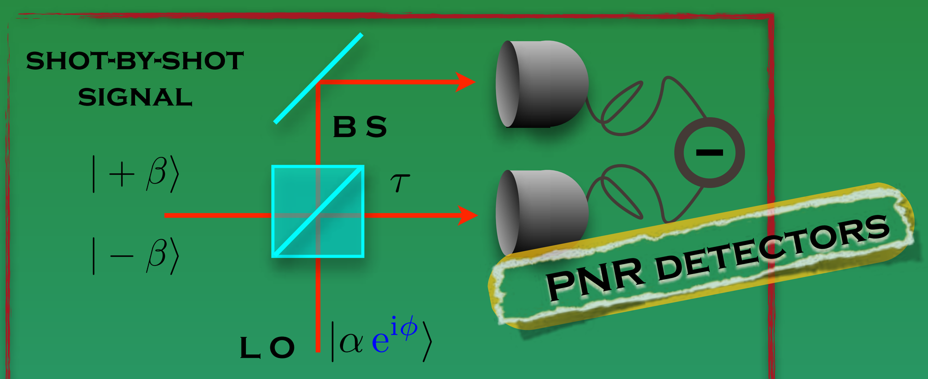


$$\mathrm{Var}_\phi \to [M\, F_\phi^{\rm PNR}]^{-1}$$

$$\mathrm{Var}_\phi \to [M\, F_\phi^{\rm on/off}]^{-1}$$

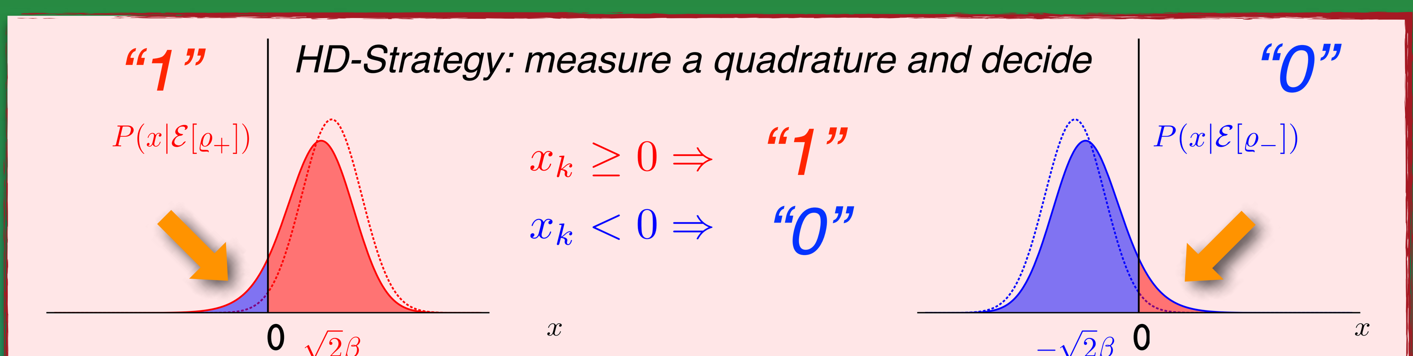


DISCRIMINATION OF NOISY COHERENT STATES: HOMODYNE-LIKE SCHEME



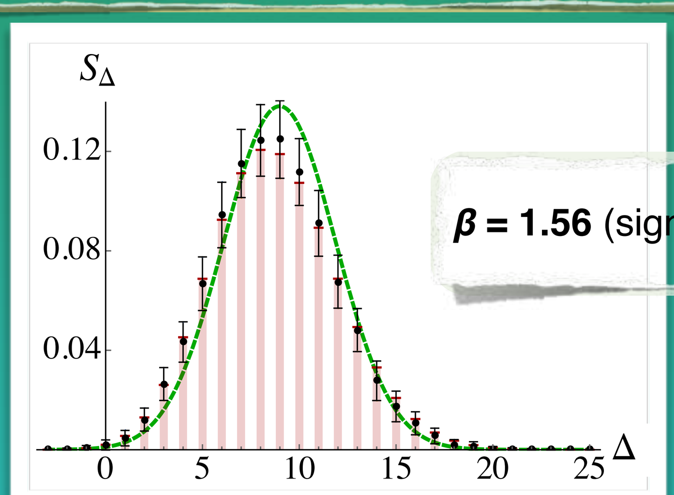
HIGH-INTENSITY LO : HOMODYNE

LOW-INTENSITY LO : HOMODYNE - LIKE

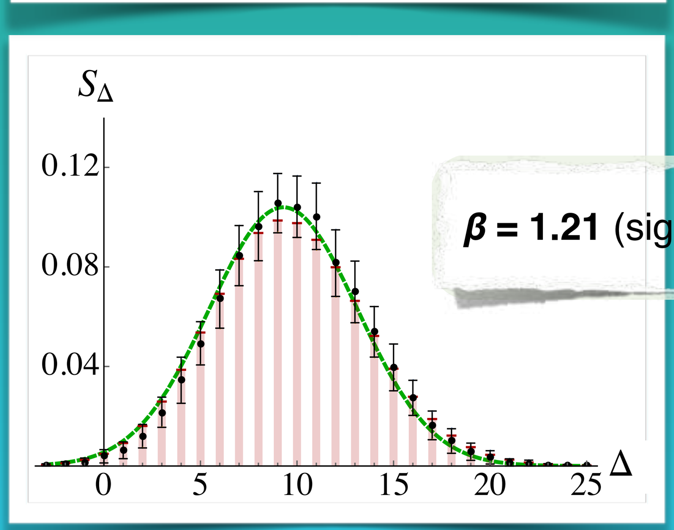


$$\mathcal{E}(\varrho_+) = \int_{\mathbb{R}} d\varphi\, f(\varphi)\hat{U}(\varphi)\varrho_+\hat{U}^\dagger(\varphi) \qquad \mathcal{E}(\varrho_-) = \int_{\mathbb{R}} d\varphi\, f(\varphi)\hat{U}(\varphi)\varrho_-\hat{U}^\dagger(\varphi)$$

difference of two Poisson distributions = Skellam distribution

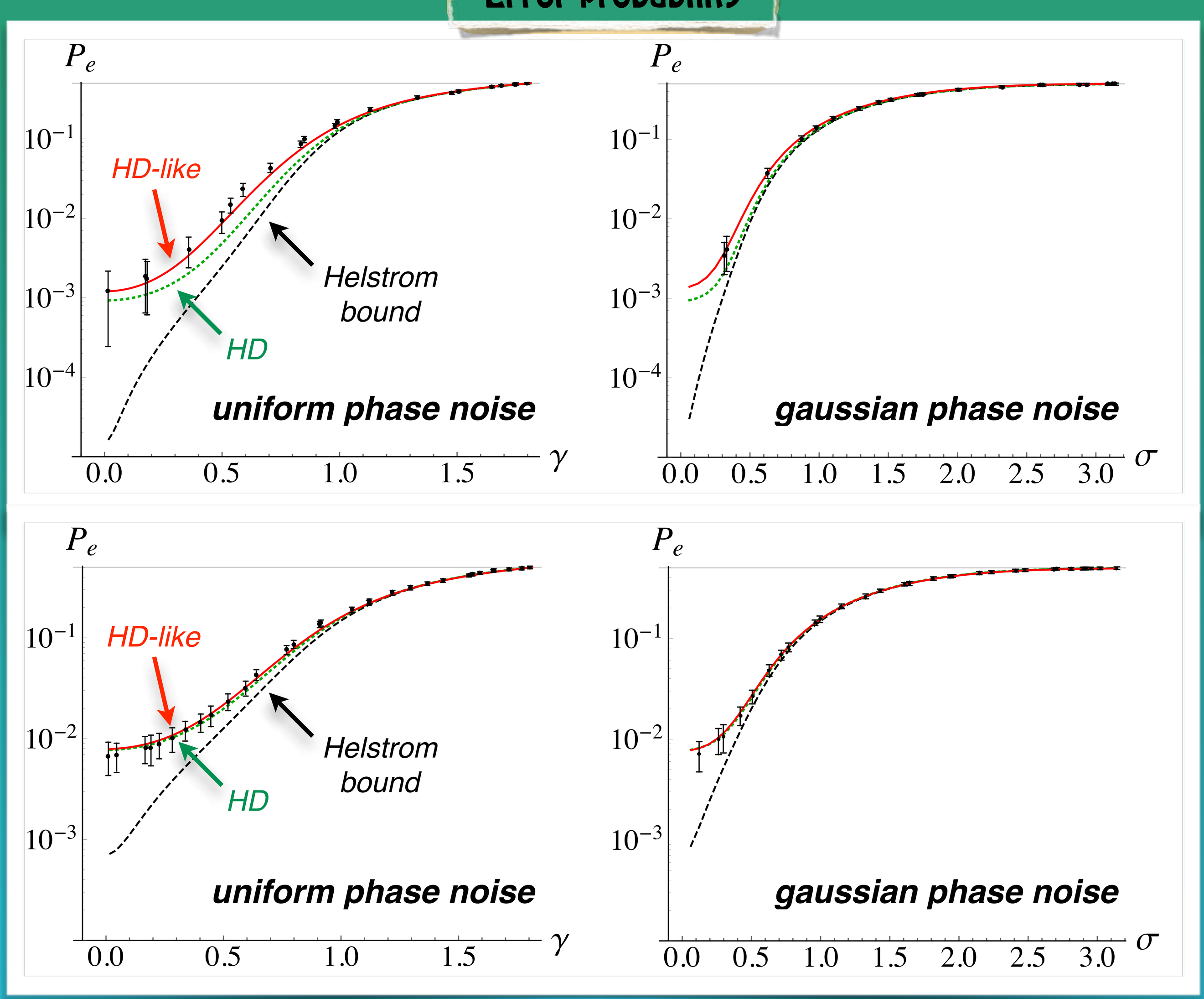


β = 1.56 (signal), α = 2.89 (LO)



β = 1.21 (signal), α = 3.84 (LO)

Error probability



DISCRIMINATION THEORY: error probability P_e

$\hat{\varrho}_0$ $\hat{\varrho}_1$: states to be distinguished

$\eta_1 + \eta_0 = 1$

HELSTROM BOUND

$$P_e^{(H)} = \frac{1}{2}\left[1 - Tr|\eta_1\hat{\rho}_1 - \eta_0\hat{\rho}_0|\right]$$

HOMODYNE-LIKE (SKELLAM) P_E

$$P_e^{(sk)} = \int_{\mathbb{R}} d\varphi f(\varphi) p_e^{(sk)}(\varphi)$$

$$p_e^{(sk)}(\varphi) = \frac{1}{2}\left[\sum_{\Delta=-\infty}^{-1} S_\Delta\big(\mu_c(\varphi), \mu_d(\varphi)\big) + \sum_{\Delta=1}^{+\infty} S_\Delta\big(\mu_c(\varphi-\pi), \mu_d(\varphi-\pi)\big) + S_0\right]$$

$$S_\Delta(\mu_c, \mu_d) = e^{-\mu_c-\mu_d}\left(\frac{\mu_c}{\mu_d}\right)^{\frac{\Delta}{2}} I_\Delta(2\sqrt{\mu_c\mu_d}), \qquad \Delta \in \mathbb{Z}$$

$$\mu_c(\phi) = a_c^2 + b_c^2 + 2a_c b_c\cos(\phi) \qquad \mu_d(\phi) = a_d^2 + b_d^2 - 2a_d b_d\cos(\phi)$$

HOMODYNE P_E

$$P_e^{(hd)} = \frac{1}{2}\left[\int_{-\infty}^{0} dx \int_{\mathbb{R}} d\varphi f(\varphi) p_{hd}(x;\beta;\varphi) + \int_0^{+\infty} dx \int_{\mathbb{R}} d\varphi f(\varphi) p_{hd}(x;-\beta;\varphi)\right]$$

$$p_{hd}(x;\pm\beta;\varphi) = \frac{1}{\sqrt{\pi}} e^{-(x\mp\sqrt{2}\beta\cos\varphi)^2}$$

CREDITS

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M. Bina, A. Allevi, M. Bondani and S. Olivares, Sci. Rep. 6, 26025 (2016)

M. Bina, A. Allevi, M. Bondani and S. Olivares, Opt. Exp. 25, 10685 (2017)

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